

Lecture 12

2025/2026

# Microwave Devices and Circuits for Radiocommunications

# 2025/2026

- 2C/1L, **MDCR**
- Attendance at minimum 7 sessions (course or laboratory)
- Lectures- **associate professor Radu Damian**
  - Tuesday **12-14, P2**
  - E – 50% final grade
  - problems + (2p atten. lect.) + (3 tests) + (bonus activity)
    - first test L1: 24.02.2026 (t2 and t3 not announced, lecture)
    - 3att.=+0.5p
  - all materials/equipments authorized

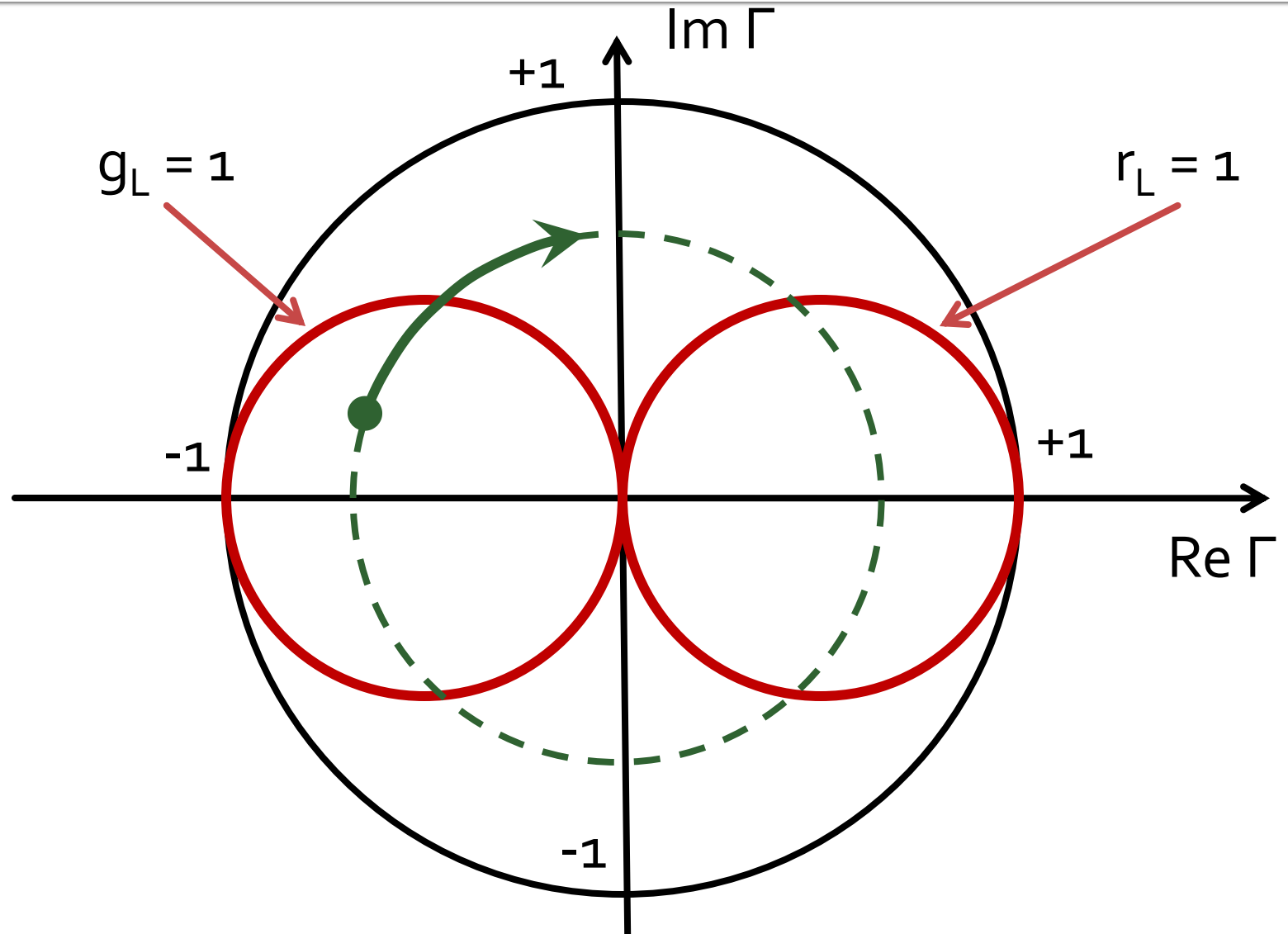
# 2025/2026

- Laboratory – **associate professor Radu Damian**
  - Monday 14-16, II.13 / (even weeks)
  - L – 25% final grade
    - ADS, 4 sessions
    - Attendance + **personal results**
  - P – 25% final grade
    - ADS, 3 sessions (-1? 24.02.2026)
    - personal homework

Impedance Matching

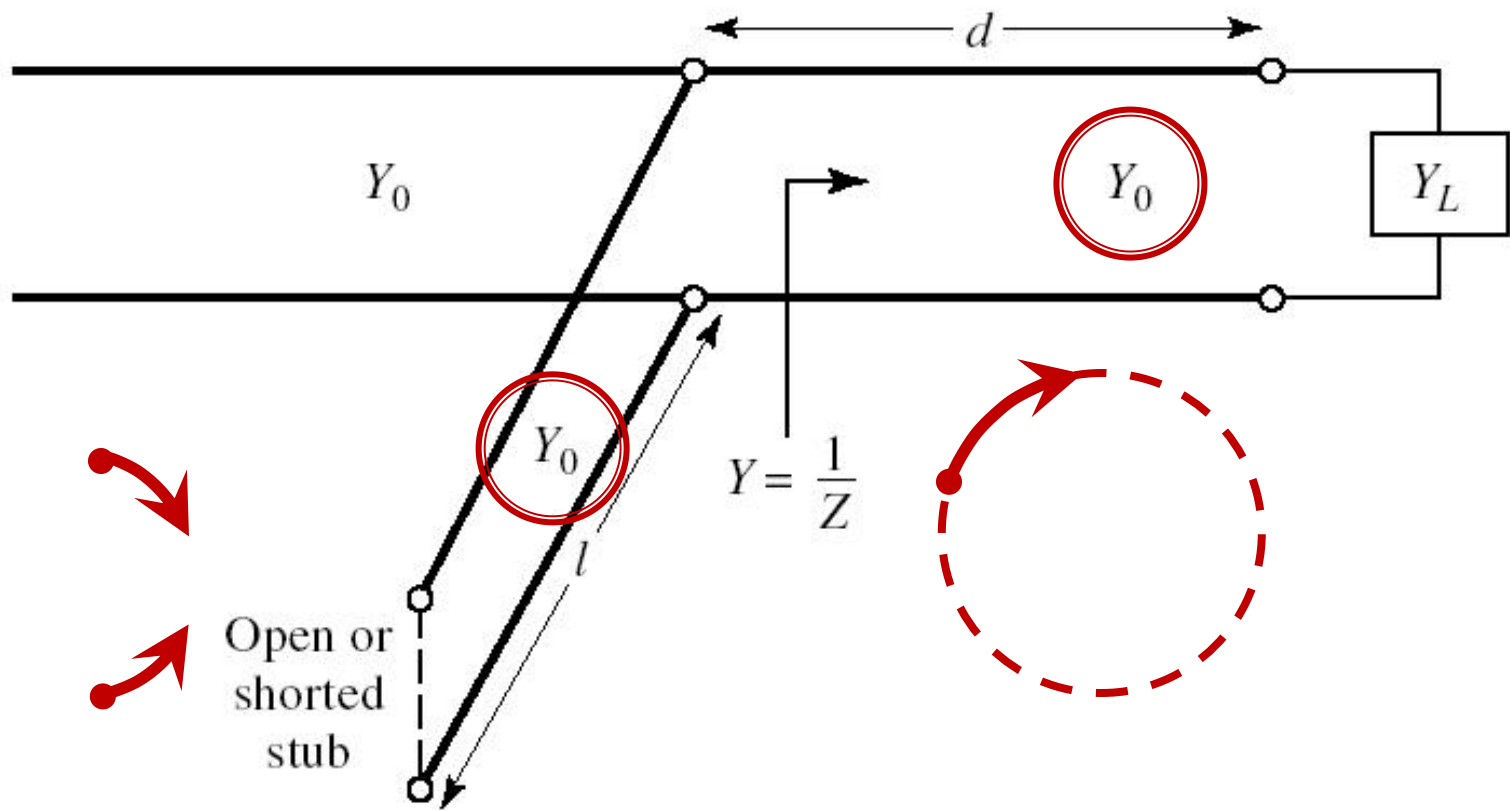
# Impedance Matching with Stubs

# Smith chart, $r=1$ and $g=1$



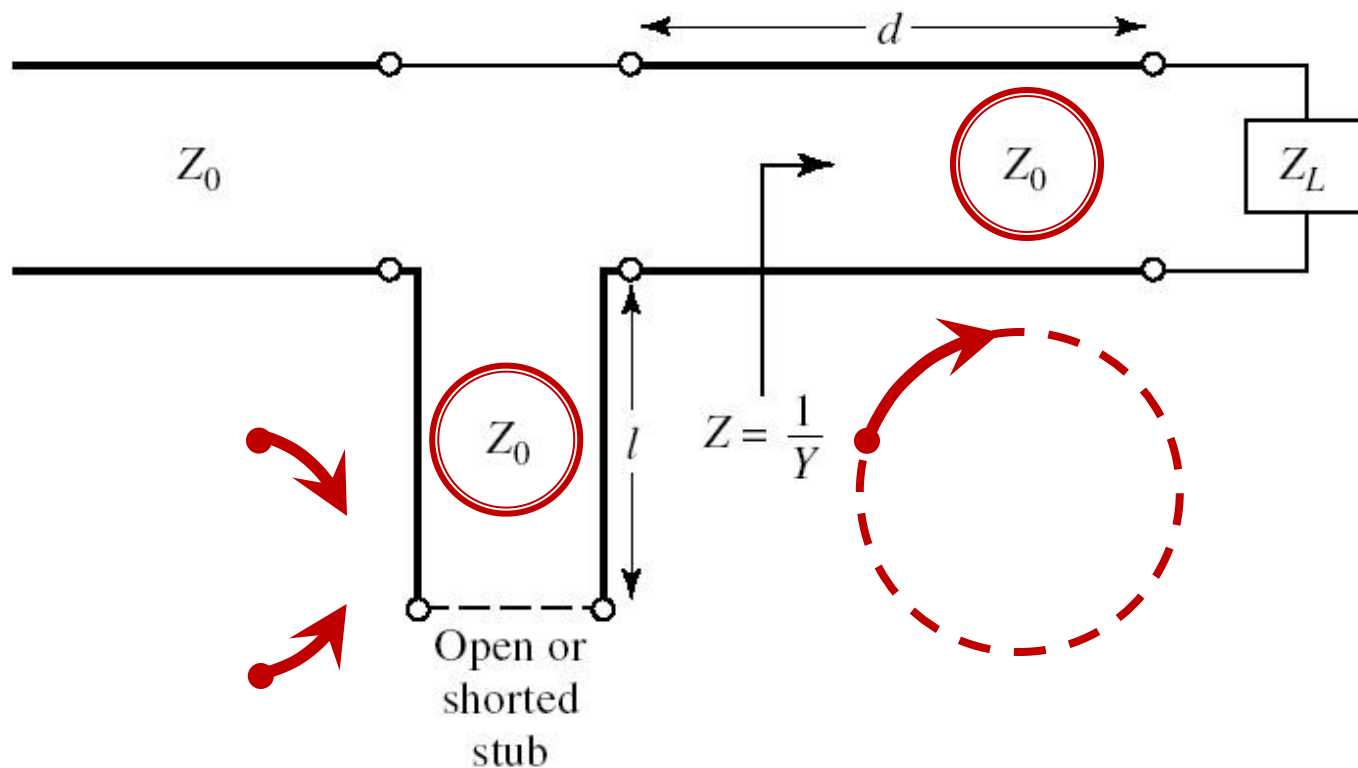
# Single stub tuning

- Shunt Stub



# Single stub tuning

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)



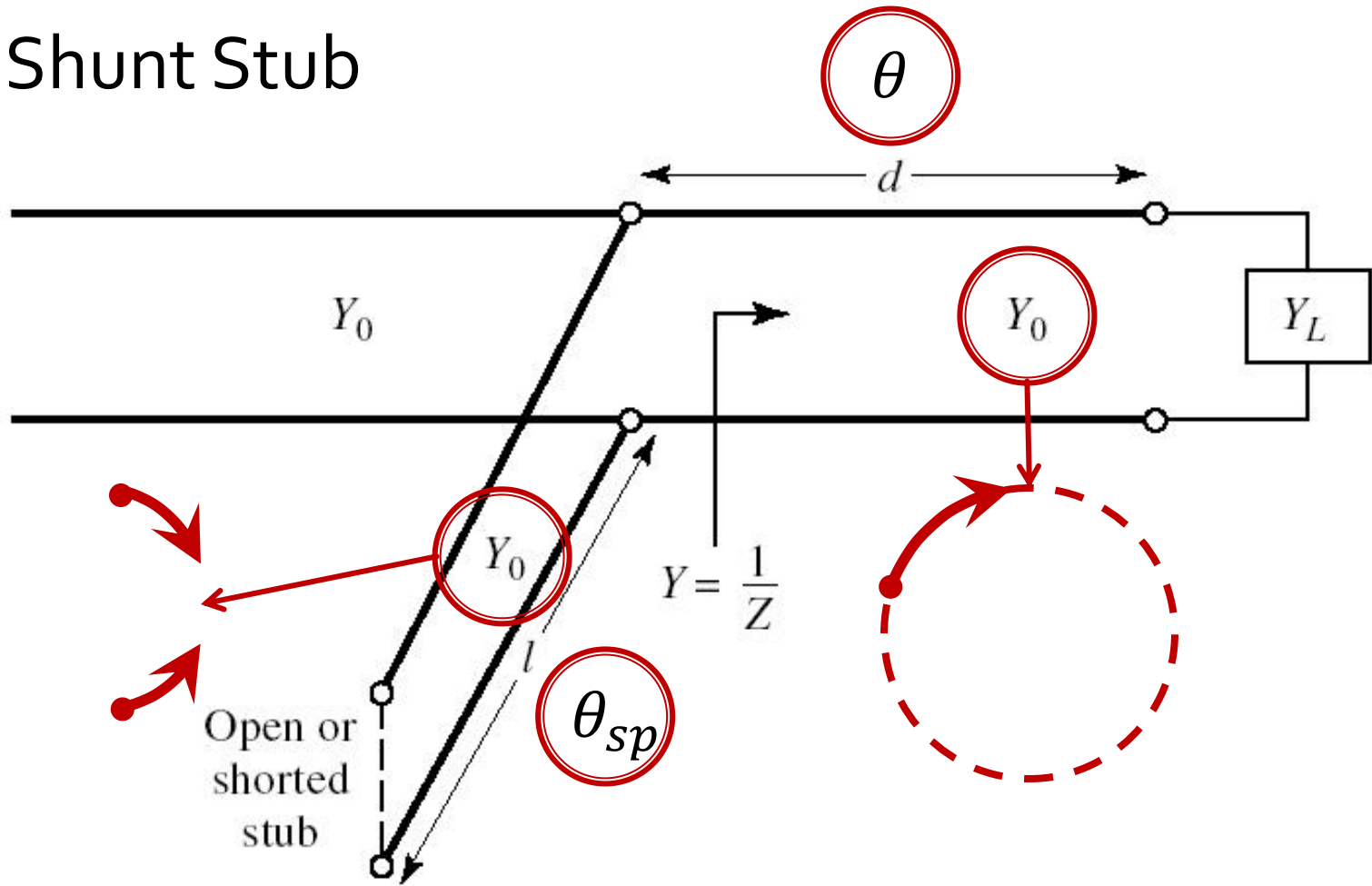
# Analytical solutions

Exam / Project

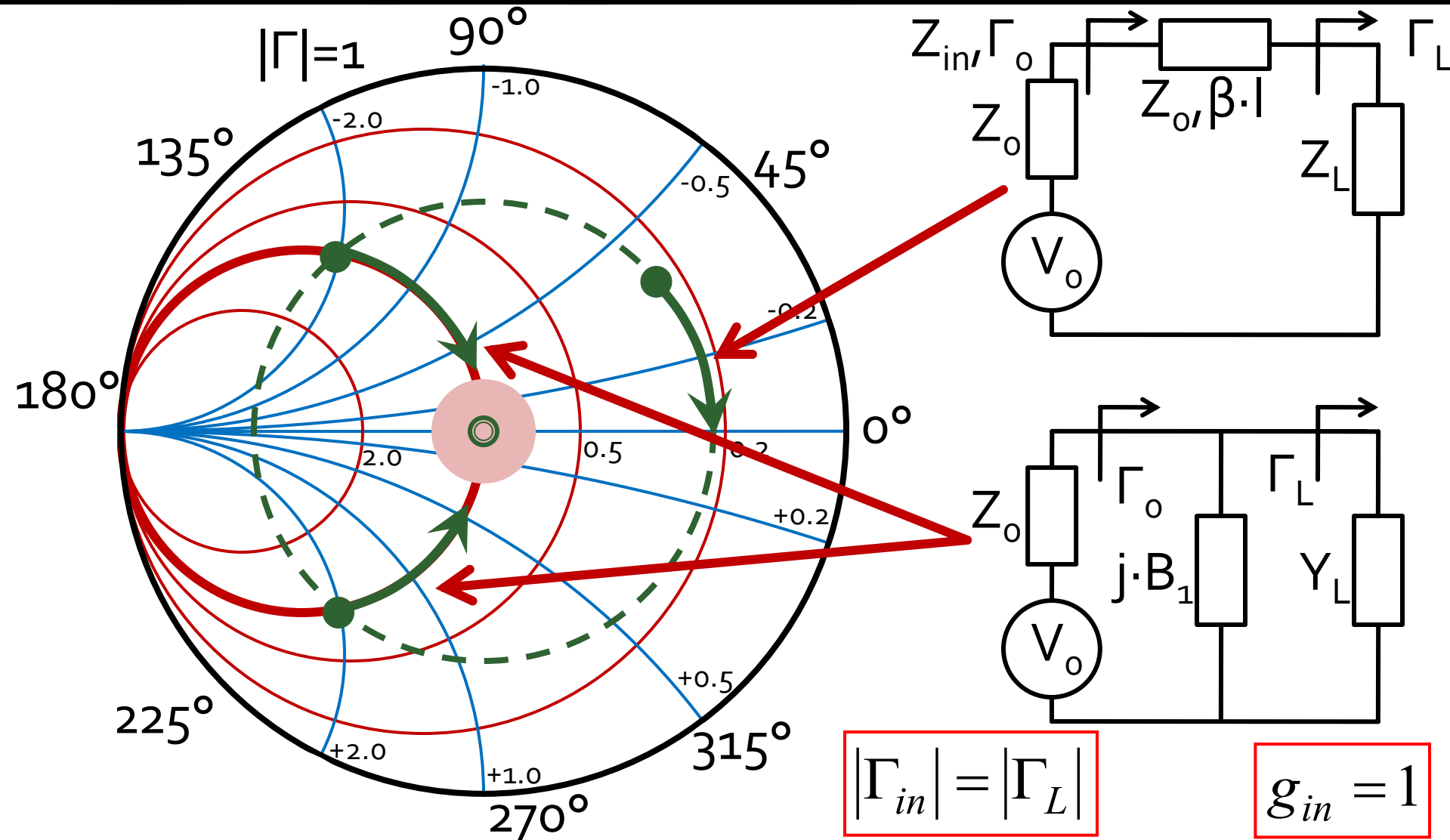
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# Case 1, Shunt Stub

- Shunt Stub



# Matching, series line + shunt susceptance



# Analytical solution, usage

$$\cos(\varphi + 2\theta) = -|\Gamma_S|$$

$$\Gamma_S = 0.593 \angle 46.85^\circ$$

$$|\Gamma_S| = 0.593; \quad \varphi = 46.85^\circ \quad \cos(\varphi + 2\theta) = -0.593 \Rightarrow (\varphi + 2\theta) = \pm 126.35^\circ$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation

- “+” solution ↓

$$(46.85^\circ + 2\theta) = +126.35^\circ \quad \theta = +39.7^\circ \quad \text{Im } y_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.472$$

$$\theta_{sp} = \tan^{-1}(\text{Im } y_S) = -55.8^\circ (+180^\circ) \rightarrow \theta_{sp} = 124.2^\circ$$

- “-” solution ↓

$$(46.85^\circ + 2\theta) = -126.35^\circ \quad \theta = -86.6^\circ (+180^\circ) \rightarrow \theta = 93.4^\circ$$

$$\text{Im } y_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.472 \quad \theta_{sp} = \tan^{-1}(\text{Im } y_S) = 55.8^\circ$$

# Analytical solution, usage

$$(\varphi + 2\theta) = \begin{cases} +126.35^\circ \\ -126.35^\circ \end{cases} \quad \theta = \begin{cases} 39.7^\circ \\ 93.4^\circ \end{cases} \quad \text{Im}[y_s(\theta)] = \begin{cases} -1.472 \\ +1.472 \end{cases} \quad \theta_{sp} = \begin{cases} -55.8^\circ + 180^\circ = 124.2^\circ \\ +55.8^\circ \end{cases}$$

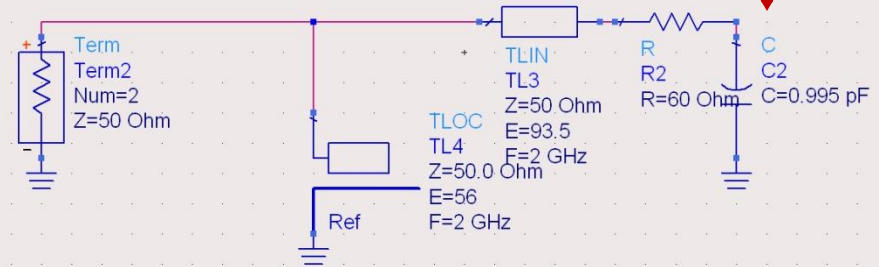
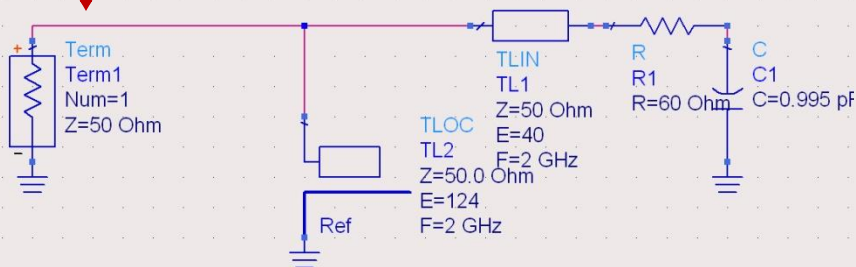
- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation

$$l_1 = \frac{39.7^\circ}{360^\circ} \cdot \lambda = 0.110 \cdot \lambda$$

$$l_2 = \frac{124.2^\circ}{360^\circ} \cdot \lambda = 0.345 \cdot \lambda$$

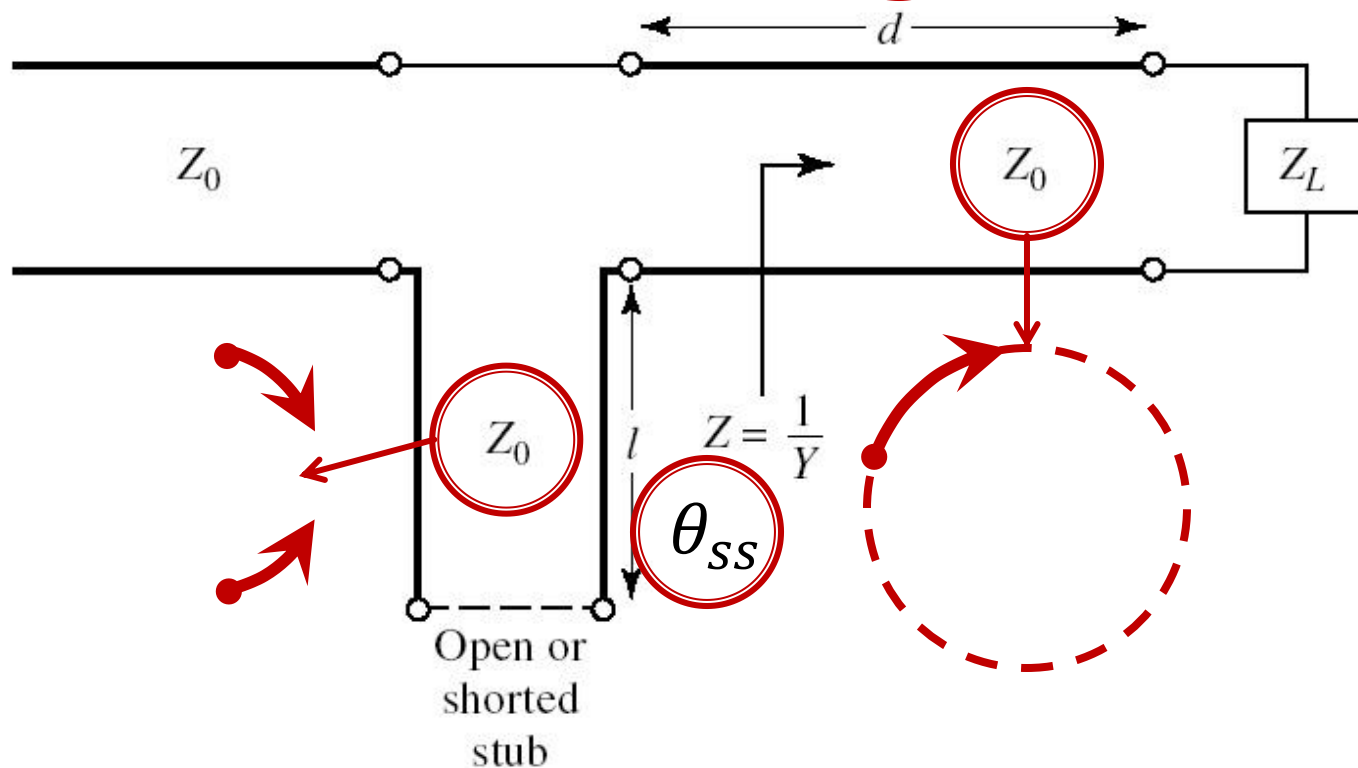
$$l_1 = \frac{93.4^\circ}{360^\circ} \cdot \lambda = 0.259 \cdot \lambda$$

$$l_2 = \frac{55.8^\circ}{360^\circ} \cdot \lambda = 0.155 \cdot \lambda$$

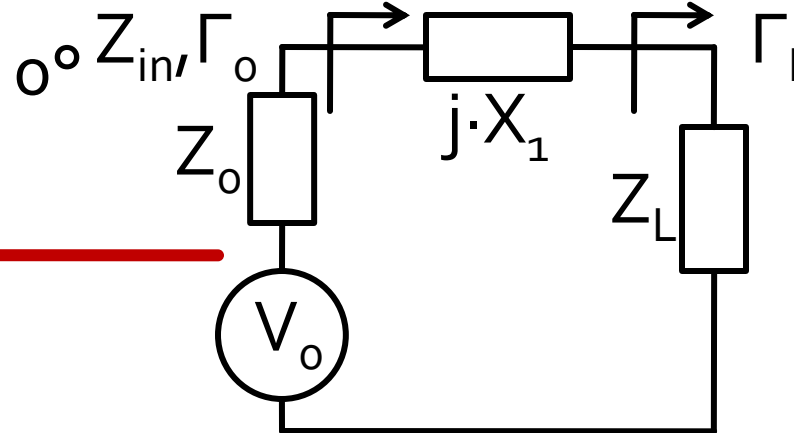
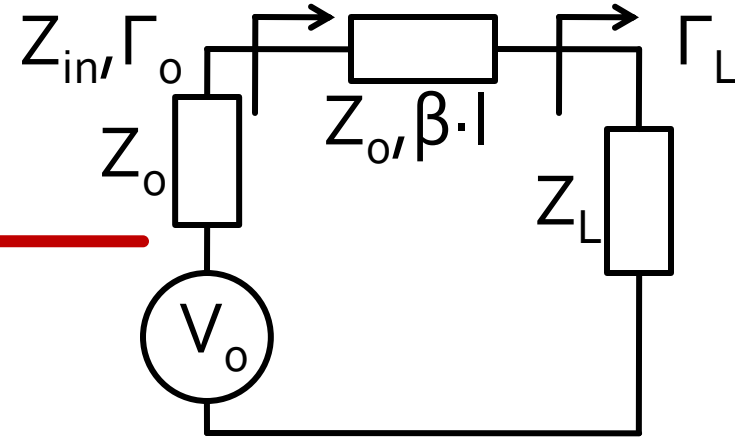
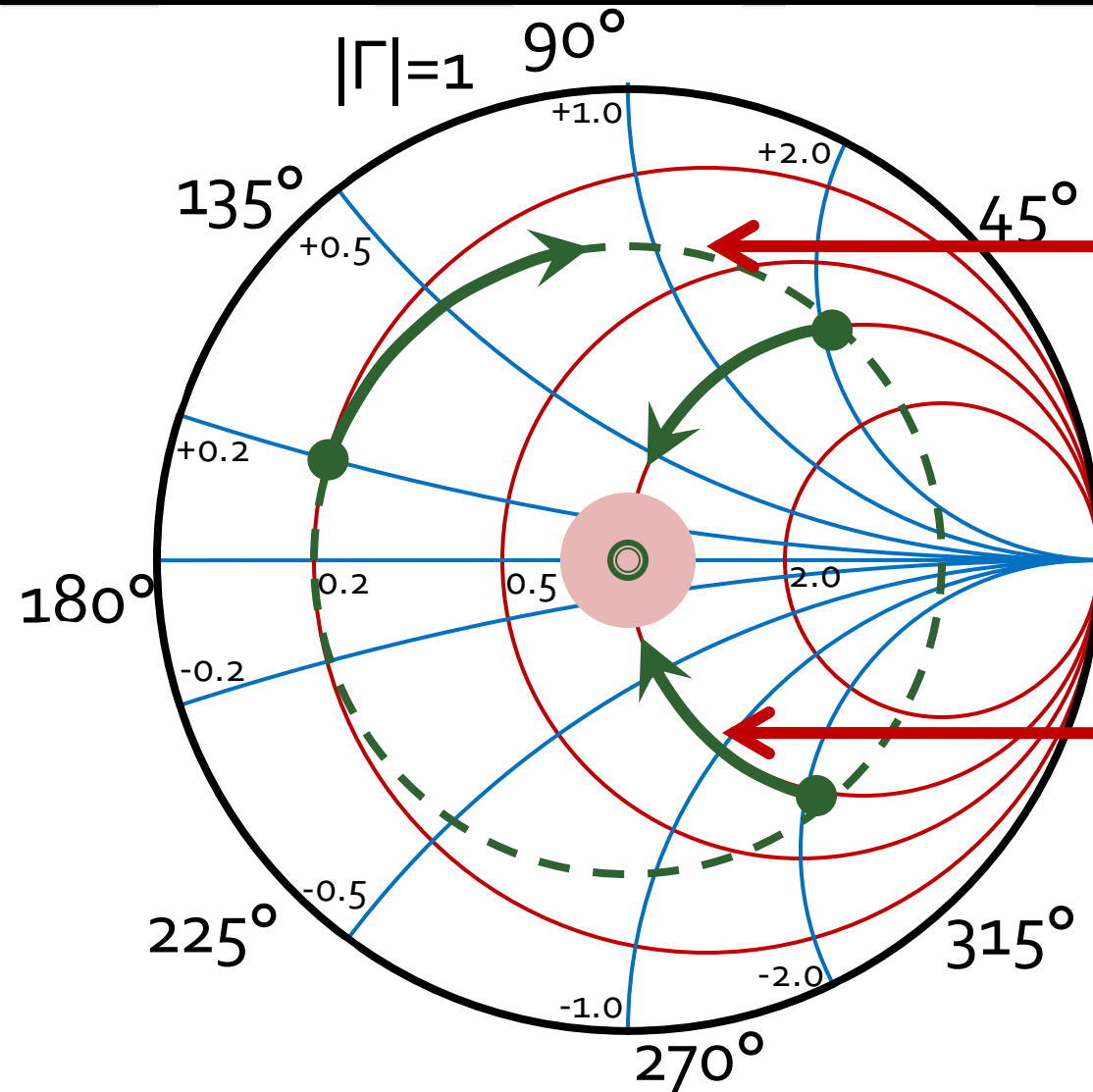


# Case 2, Series Stub

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)  $\theta$



# Matching, series line + series reactance



$$|\Gamma_{in}| = |\Gamma_L|$$

$$r_{in} = 1$$

# Analytical solution, usage

$$\cos(\varphi + 2\theta) = |\Gamma_S|$$

$$\theta_{ss} = \beta \cdot l = \cot^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\Gamma_S = 0.555 \angle -29.92^\circ$$

$$|\Gamma_S| = 0.555; \quad \varphi = -29.92^\circ \quad \cos(\varphi + 2\theta) = 0.555 \Rightarrow (\varphi + 2\theta) = \pm 56.28^\circ$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation

- **"+" solution** ↓

$$(-29.92^\circ + 2\theta) = +56.28^\circ \quad \theta = 43.1^\circ \quad \text{Im } z_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.335$$

$$\theta_{ss} = -\cot^{-1}(\text{Im } z_S) = -36.8^\circ (+180^\circ) \rightarrow \theta_{ss} = 143.2^\circ$$

- **"-" solution** ↓

$$(-29.92^\circ + 2\theta) = -56.28^\circ \quad \theta = -13.2^\circ (+180^\circ) \rightarrow \theta = 166.8^\circ$$

$$\text{Im } z_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.335 \quad \theta_{ss} = -\cot^{-1}(\text{Im } z_S) = 36.8^\circ$$

# Analytical solution, usage

$$(\varphi + 2\theta) = \begin{cases} +56.28^\circ \\ -56.28^\circ \end{cases} \quad \theta = \begin{cases} 43.1^\circ \\ 166.8^\circ \end{cases} \quad \text{Im}[z_s(\theta)] = \begin{cases} +1.335 \\ -1.335 \end{cases} \quad \theta_{ss} = \begin{cases} -36.8^\circ + 180^\circ = 143.2^\circ \\ +36.8^\circ \end{cases}$$

- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation

$$l_1 = \frac{43.1^\circ}{360^\circ} \cdot \lambda = 0.120 \cdot \lambda$$

$$l_2 = \frac{143.2^\circ}{360^\circ} \cdot \lambda = 0.398 \cdot \lambda$$

$$l_1 = \frac{166.8^\circ}{360^\circ} \cdot \lambda = 0.463 \cdot \lambda$$

$$l_2 = \frac{36.8^\circ}{360^\circ} \cdot \lambda = 0.102 \cdot \lambda$$



# Stub, observations

- adding or subtracting **180°** ( $\lambda/2$ ) doesn't change the result (full rotation around the Smith Chart)

$$E = \beta \cdot l = \pi = 180^\circ \quad l = k \cdot \frac{\lambda}{2}, \forall k \in \mathbf{N}$$

- if the lines/stubs result with **negative** "length"/ "electrical length" we add  $\lambda/2$  /  $180^\circ$  to obtain physically realizable lines
- adding or subtracting **90°** ( $\lambda/4$ ) change the stub impedance:

$$Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l \quad \Leftrightarrow \quad Z_{in,g} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

- for the stub we can add or subtract  $90^\circ$  ( $\lambda/4$ ) while in the same time changing **open-circuit**  $\Leftrightarrow$  **short-circuit**

# Microwave Filters

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# Insertion loss method

- We control the power loss ratio/attenuation introduced by the filter:
  - in the passband (pass all frequencies)
  - in the stopband (reject all frequencies)

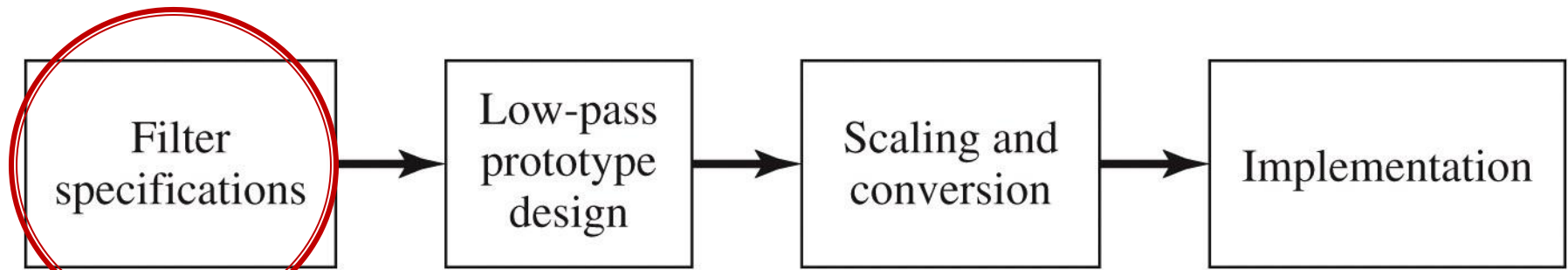
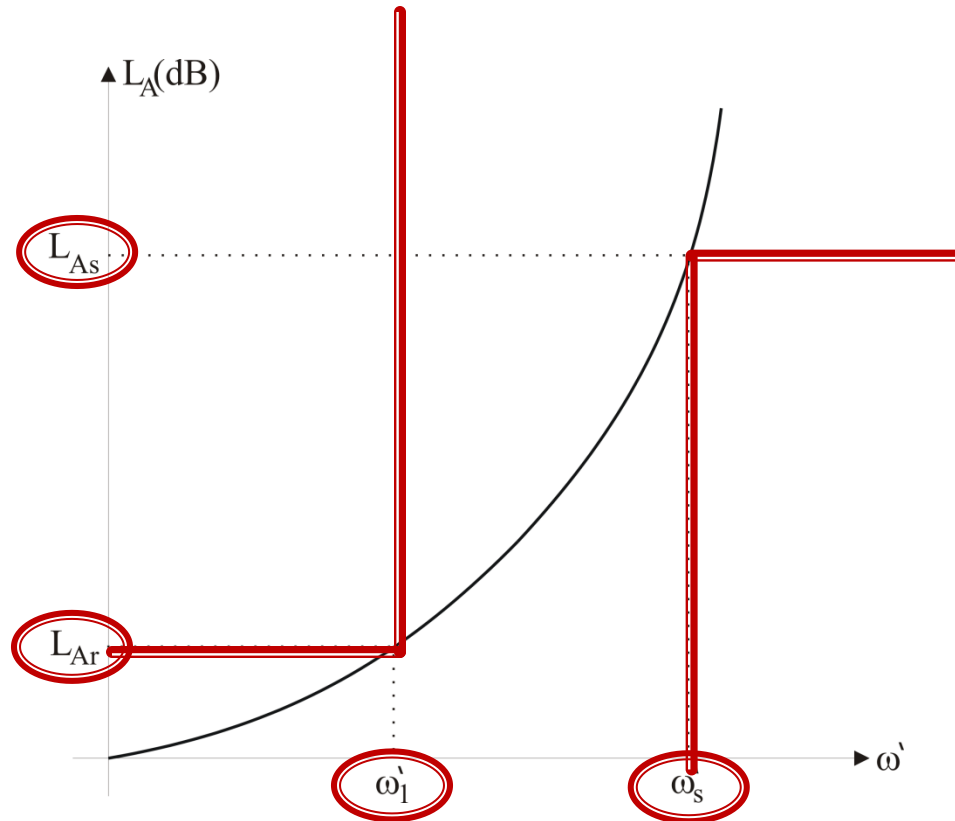


Figure 8.23

# Filter specifications

- Attenuation
  - in passband
  - in stopband
  - most often in **dB**
- Frequency range
  - passband
  - stopband
  - cutoff frequency  $\omega_1'$   
usually normalized  
(= **1**)



# Insertion loss method

- We choose the right polynomials to design an **low-pass** filter (prototype)
- The low-pass prototype are then converted to the desired other types of filters
  - low-pass, high-pass, bandpass, or bandstop

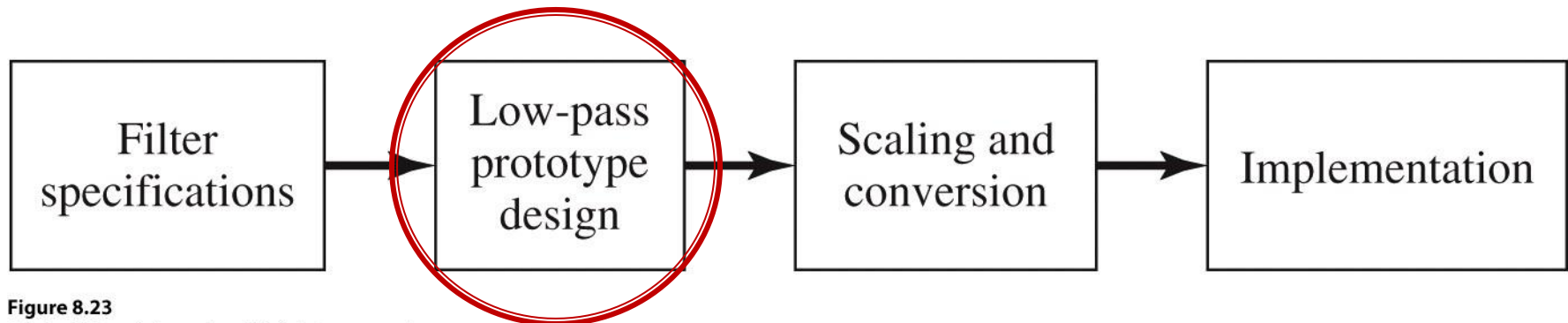


Figure 8.23

# Maximally Flat/Equal ripple LPF Prototype

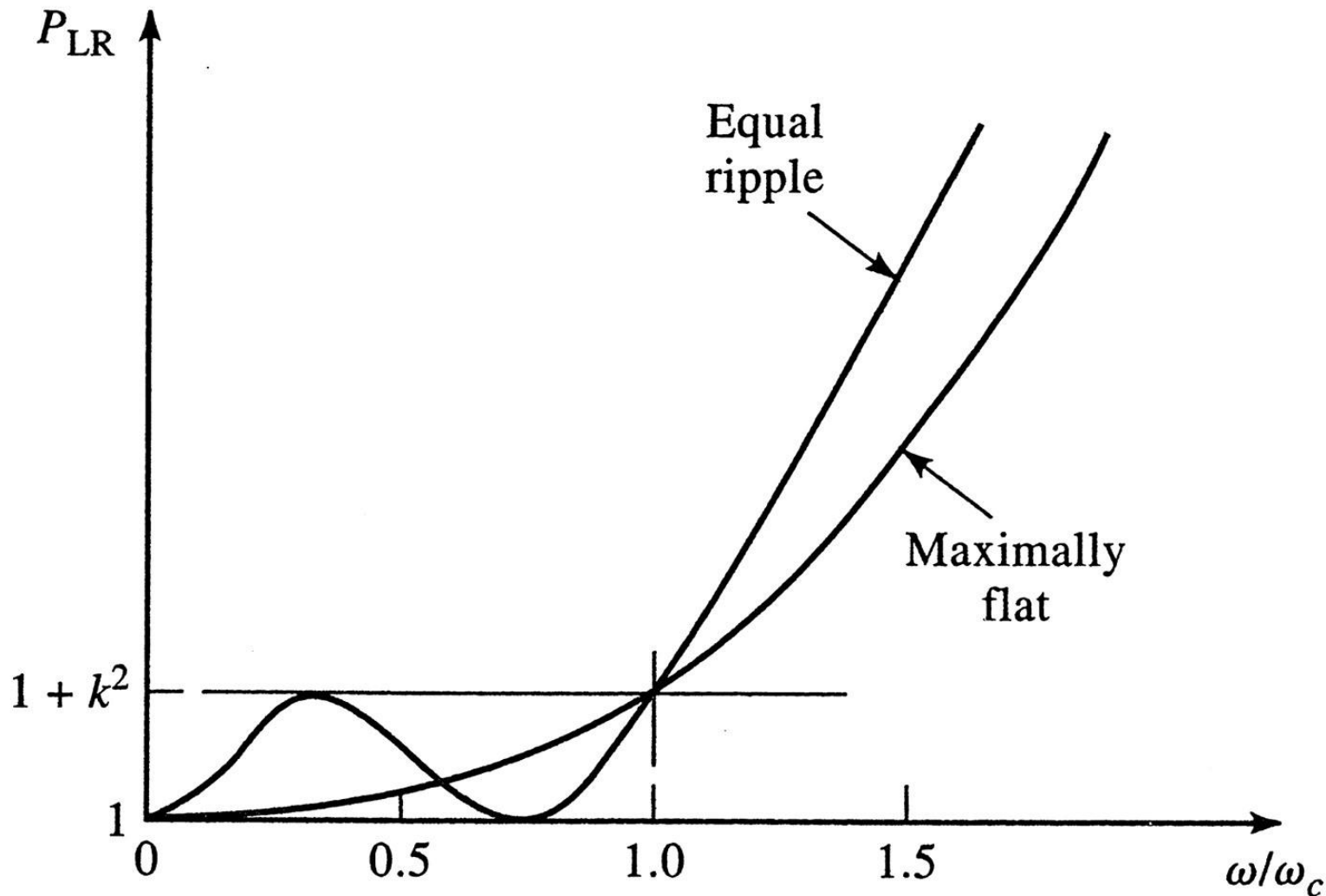
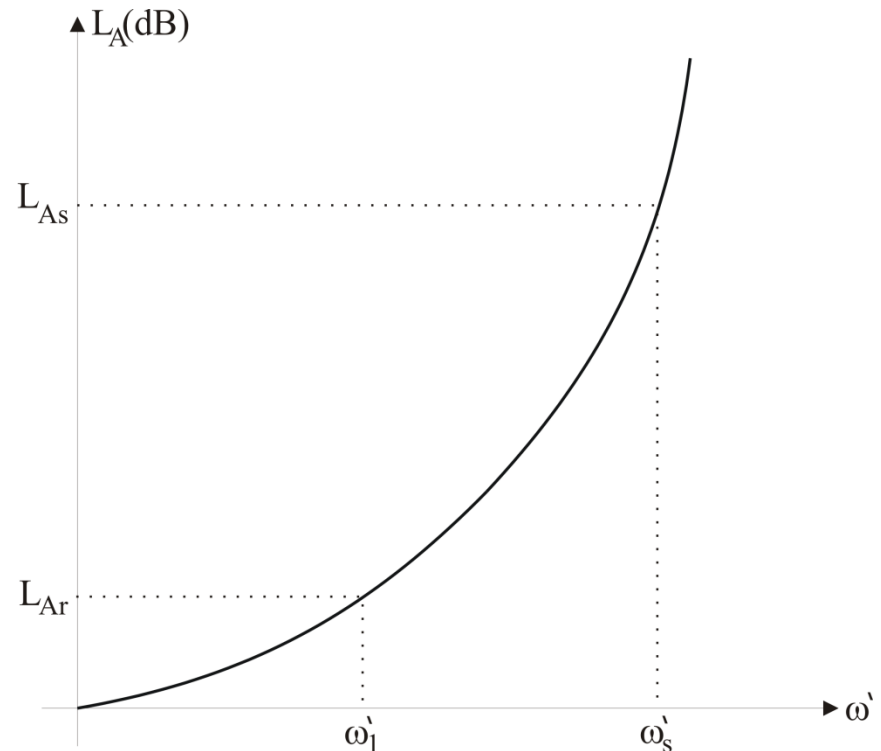


Figure 8.21  
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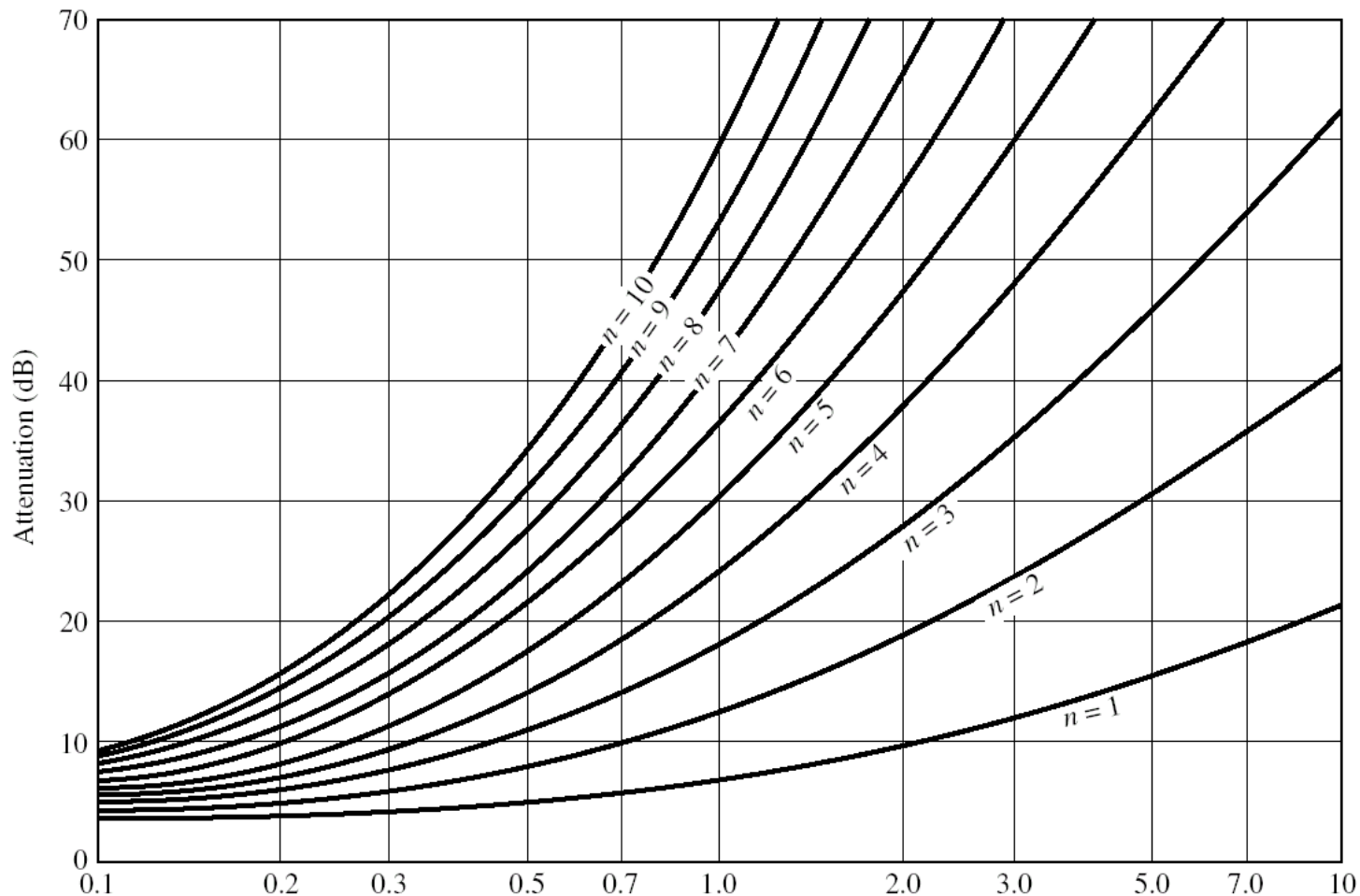
# Order (N) of the Maximally Flat filter

$$n \geq \frac{\log \left( \frac{10^{\frac{L_{As}}{10}} - 1}{10^{\frac{L_{Ar}}{10}} - 1} \right)}{2 \cdot \log \frac{\omega'_s}{\omega'_1}}$$

- !attenuations in **dB**



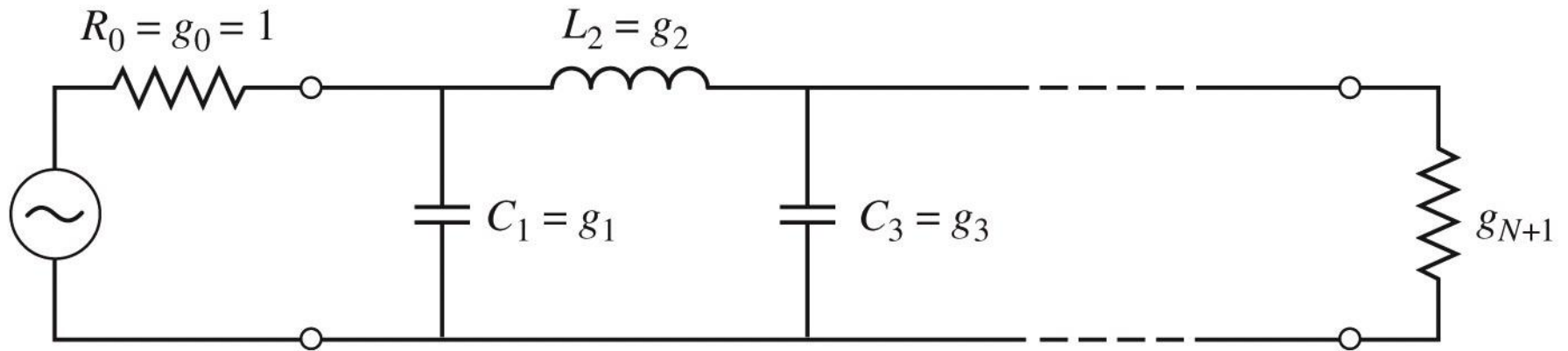
# Maximally flat filter prototypes



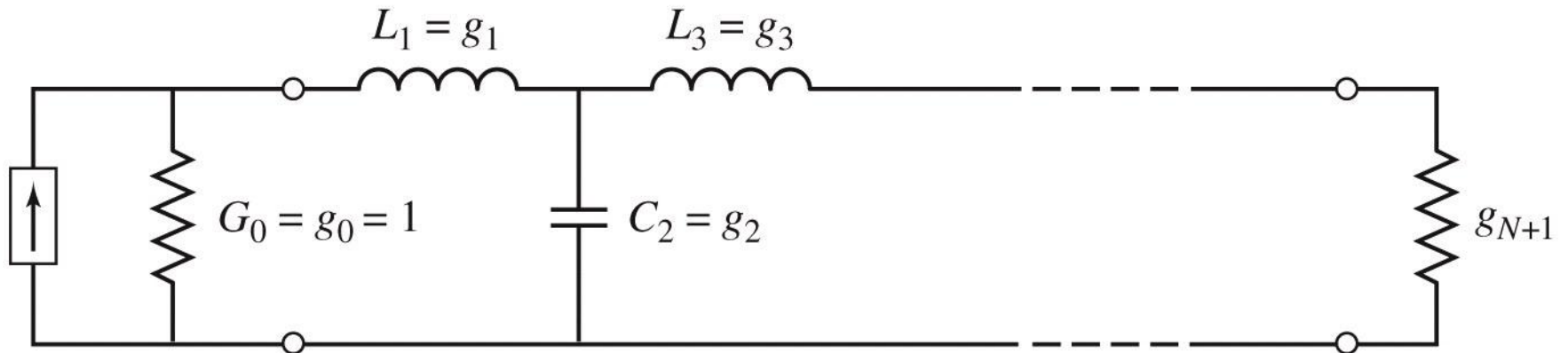
$$\left| \frac{\omega}{\omega_c} \right| - 1$$

Attenuation versus normalized frequency for maximally flat filter prototypes

# Prototype Filters



(a)



(b)

# Prototype Filters

- Prototype filters are:
  - Low-Pass Filters (**LPF**)
  - cutoff frequency  **$\omega_o = 1 \text{ rad/s}$**  ( $f_o = 0.159 \text{ Hz}$ )
  - connected to a source with  **$R = 1\Omega$**
- The number of reactive elements (L/C) is the order of the filter (N)
- Reactive elements are alternated: series L / shunt C
- There two prototypes with the same response, a prototype beginning with a shunt C element, and a prototype beginning with a series L element

# Prototype Filters

- We define filter parameters  $g_i$ ,  $i=0, N+1$
- $g_i$  are the element values in the prototype filter

$$g_0 = \begin{cases} \text{generator resistance } R'_0 & \text{if } g_1 = C'_1 \\ \text{generator conductance } G'_0 & \text{if } g_1 = L'_1 \end{cases}$$

$$g_k \Big|_{k=1, \overline{N}} = \begin{cases} \text{inductance for series inductors} \\ \text{capacitance for shunt capacitors} \end{cases}$$

$$g_{N+1} = \begin{cases} \text{load resistance } R'_{N+1} & \text{if } g_N = C'_N \\ \text{load conductance } G'_{N+1} & \text{if } g_N = L'_N \end{cases}$$

# Maximally Flat LPF Prototype

- Formulas for filter parameters

$$g_0 = 1$$

$$g_k = 2 \cdot \sin \left[ \frac{(2 \cdot k - 1) \cdot \pi}{2 \cdot N} \right], \quad k = 1, N$$

$$g_{N+1} = 1$$

# Maximally Flat LPF Prototype

**TABLE 8.3** Element Values for Maximally Flat Low-Pass Filter Prototypes ( $g_0 = 1$ ,  $\omega_c = 1$ ,  $N = 1$  to 10)

$N$	$g_1$	$g_2$	$g_3$	$g_4$	$g_5$	$g_6$	$g_7$	$g_8$	$g_9$	$g_{10}$	$g_{11}$
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

*Source:* Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

**TABLE 8.4** Element Values for Equal-Ripple Low-Pass Filter Prototypes ( $g_0 = 1, \omega_c = 1, N = 1$  to 10, 0.5 dB and 3.0 dB ripple)

0.5 dB Ripple											
$N$	$g_1$	$g_2$	$g_3$	$g_4$	$g_5$	$g_6$	$g_7$	$g_8$	$g_9$	$g_{10}$	$g_{11}$
1	0.6986	1.0000									
2	1.4029	0.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.8696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.0000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7239	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5239	0.8842	1.9841

3.0 dB Ripple											
$N$	$g_1$	$g_2$	$g_3$	$g_4$	$g_5$	$g_6$	$g_7$	$g_8$	$g_9$	$g_{10}$	$g_{11}$
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

- For even N order of the filter (N = 2, 4, 6, 8 ...) equal-ripple filters **must** closed by a load impedance

$g_{N+1} \neq 1$

- If the application doesn't allow this, supplemental impedance matching is required (quarter-wave transformer, binomial ...) to  $g_L = 1$

# Impedance and Frequency Scaling

- LPF Prototype is only used as an intermediate step
  - Low-Pass Filter (LPF)
  - cutoff frequency  $\omega_o = 1 \text{ rad/s}$  ( $f_o = 0.159 \text{ Hz}$ )
  - connected to a source with  $R = 1\Omega$

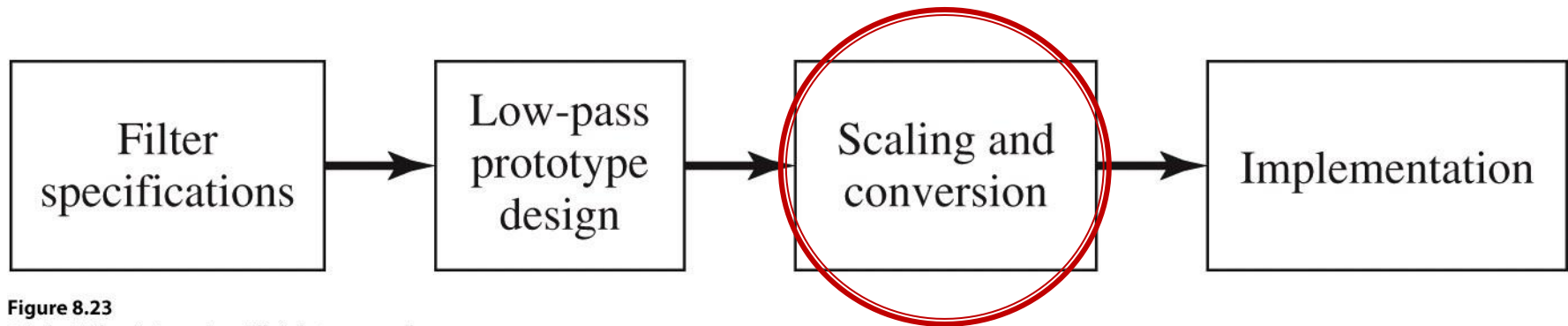


Figure 8.23

# Impedance Scaling

- To design a filter which will work with a source resistance of  $R_0$  we multiply all the impedances of the prototype design by  $R_0$  (" $'$ " denotes scaled values)

$$R'_s = R_0 \cdot (R_s = 1)$$

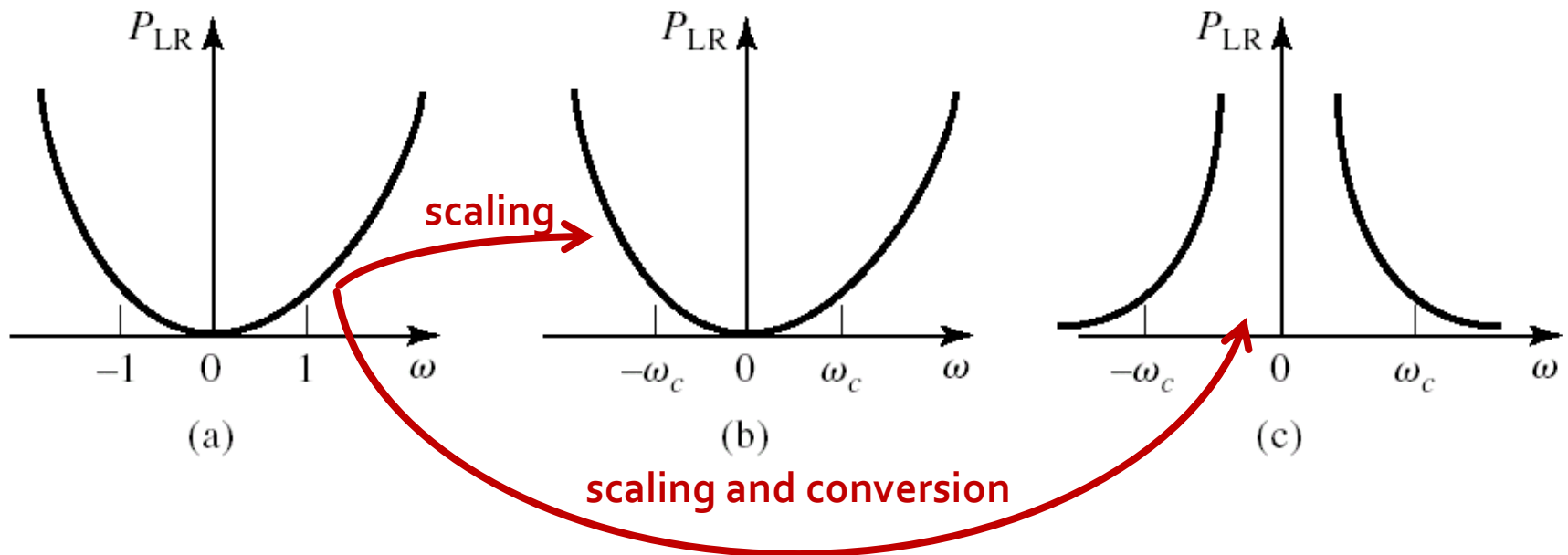
$$R'_L = R_0 \cdot R_L$$

$$L' = R_0 \cdot L$$


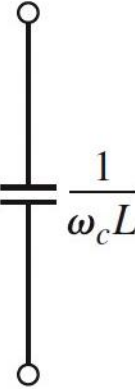
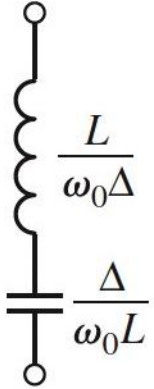
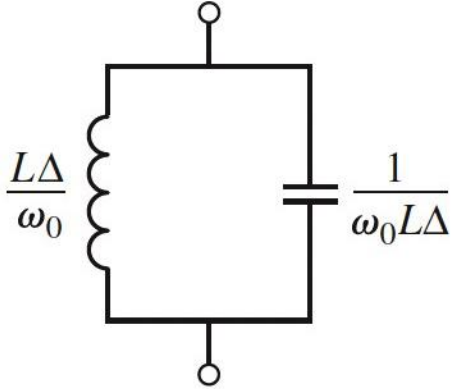
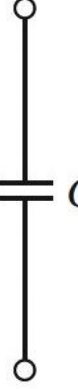
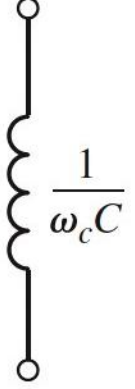
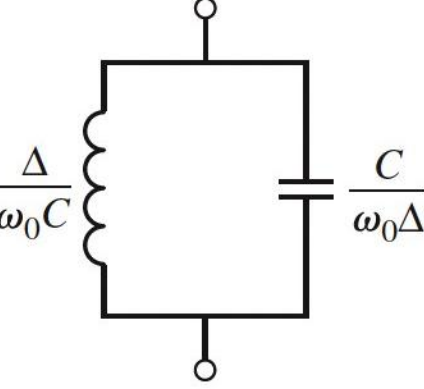
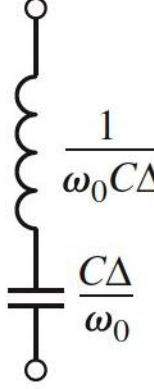
$$C' = \frac{C}{R_0}$$

# Frequency Scaling

- changing the cutoff frequency – (fig. b)
- changing the type (for example LPF  $\rightarrow$  HPF – fig. c) requires also conversion



# Summary of Prototype Filter Transformations

Low-pass	High-pass	Bandpass	Bandstop
 <p style="text-align: center;"><math>L</math></p>	 <p style="text-align: center;"><math>\frac{1}{\omega_c L}</math></p>	 <p style="text-align: center;"><math>\frac{L}{\omega_0 \Delta}</math> <math>\frac{\Delta}{\omega_0 L}</math></p>	 <p style="text-align: center;"><math>\frac{L \Delta}{\omega_0}</math>      <math>\frac{1}{\omega_0 L \Delta}</math></p>
 <p style="text-align: center;"><math>C</math></p>	 <p style="text-align: center;"><math>\frac{1}{\omega_c C}</math></p>	 <p style="text-align: center;"><math>\frac{\Delta}{\omega_0 C}</math>      <math>\frac{C}{\omega_0 \Delta}</math></p>	 <p style="text-align: center;"><math>\frac{1}{\omega_0 C \Delta}</math> <math>\frac{C \Delta}{\omega_0}</math></p>

# Microwave Filters Implementation

- The lumped-element (L, C) filter design generally works well **only** at low frequencies (RF):
  - lumped-element inductors and capacitors are generally available only for a limited range of values, and can be difficult to implement at microwave frequencies
  - difficulty to obtain the (very low) required tolerance for elements

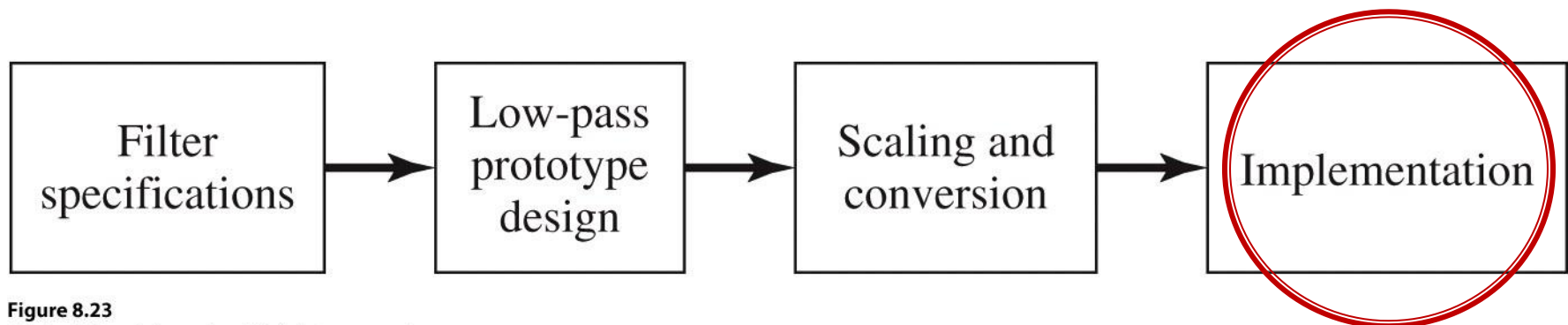
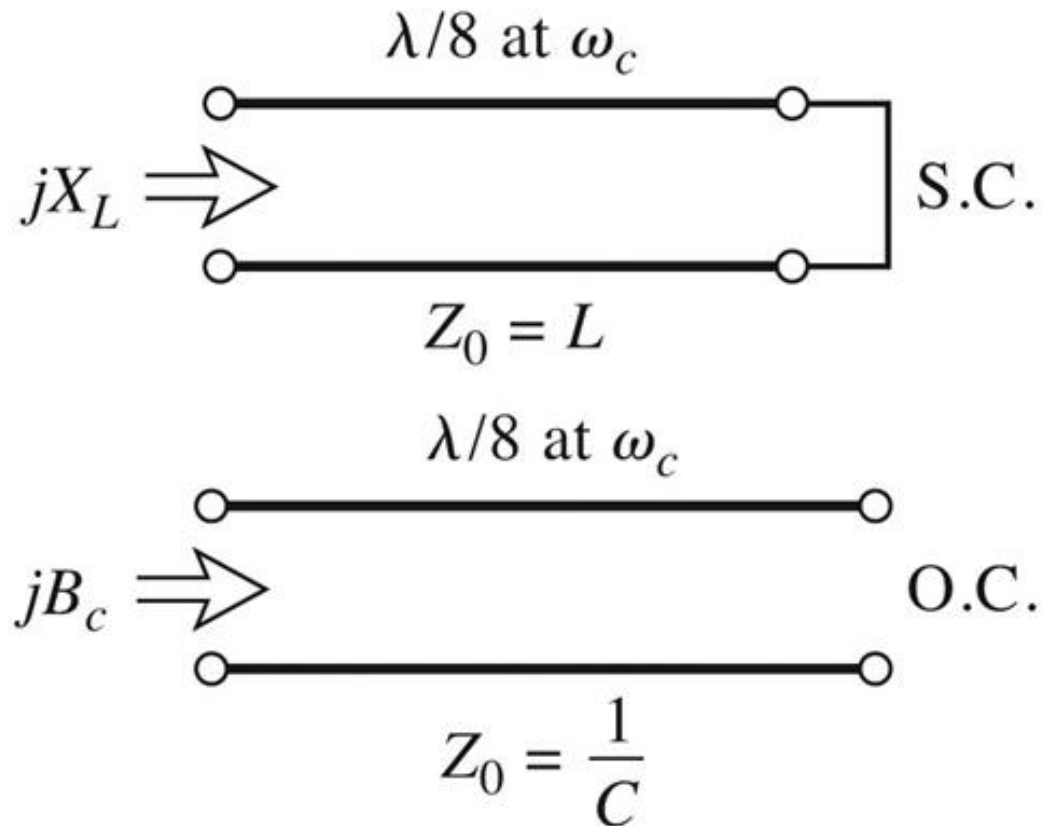
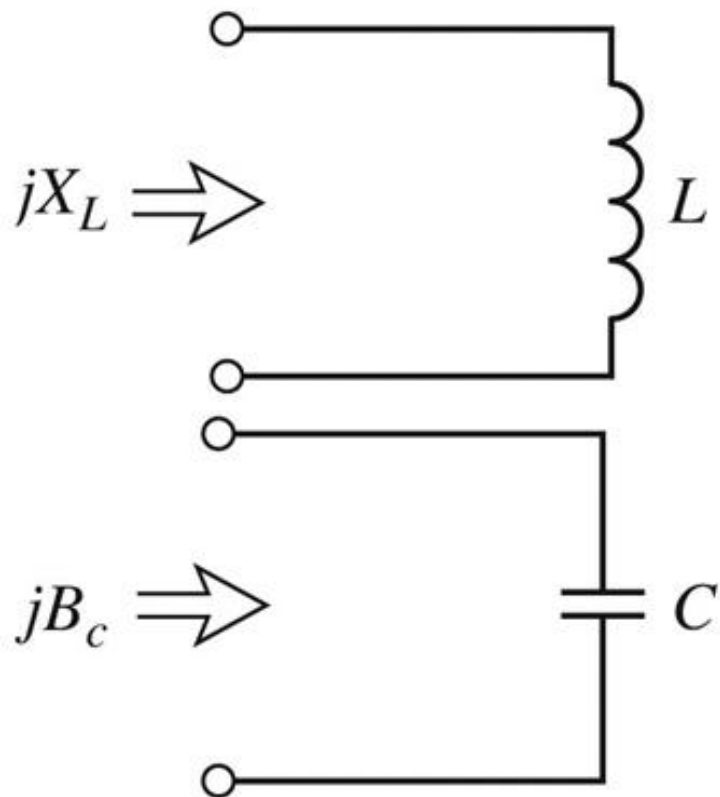


Figure 8.23

# Richards' Transformation

- allows implementation of the inductors and capacitors with lines **after** the transformation of the LPF prototype to the required type (LPF/HPF/BPF/BSF)



# Richards' Transformation

- the periodicity of tan function implies the periodicity of the filter implemented with lines
  - the filter response will be repeated every  $4 \cdot \omega_c$

$$\tan(\alpha + \pi) = \tan \alpha$$

$$\beta \cdot l \Big|_{\omega=\omega_c} = \frac{\pi}{4} \Rightarrow \frac{\omega_c \cdot l}{v_p} = \frac{\pi}{4} \Rightarrow \pi = \frac{(4 \cdot \omega_c) \cdot l}{v_p}$$

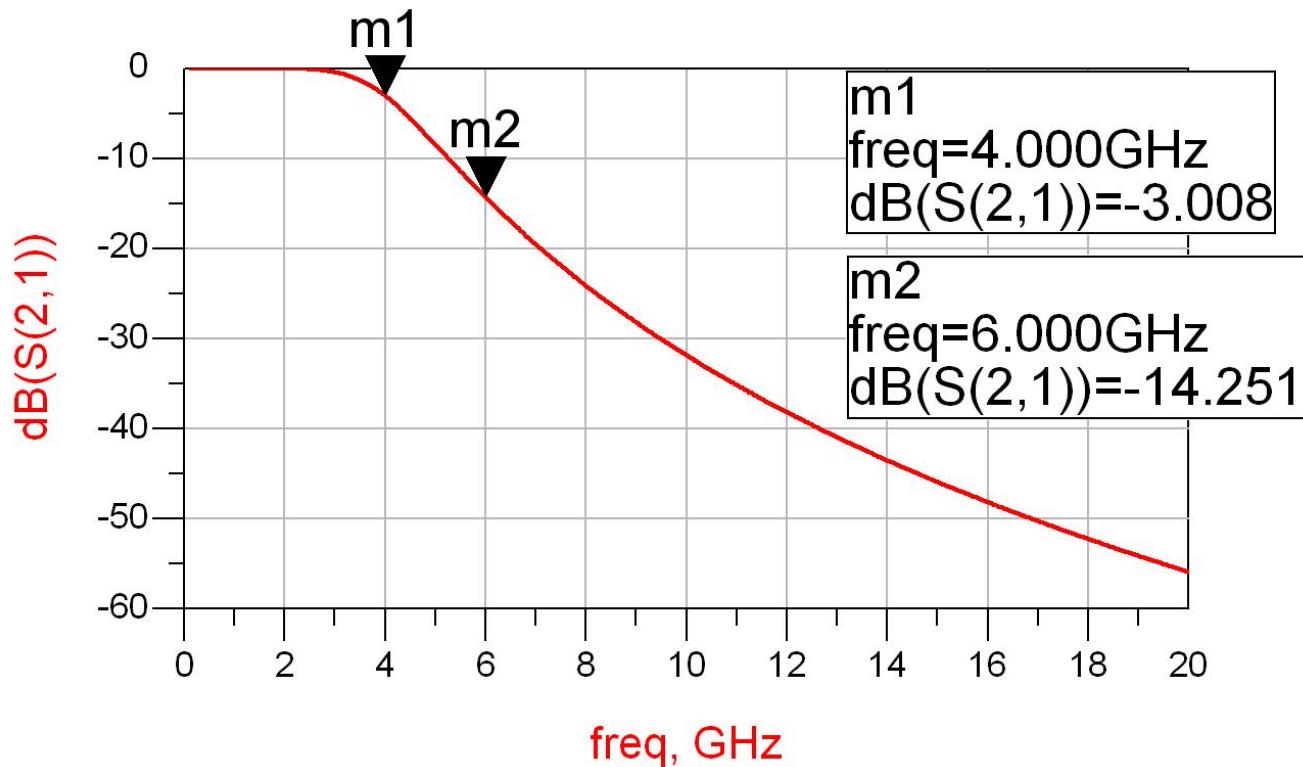
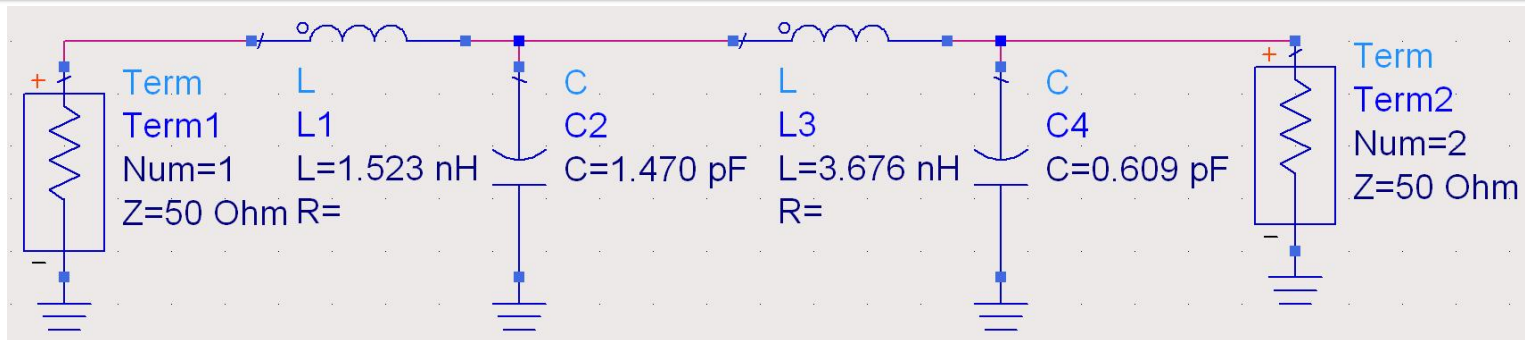
$$Z_{in}(\omega) = Z_{in}(\omega + 4 \cdot \omega_c) \Rightarrow P_{LR}(\omega) = P_{LR}(\omega + 4 \cdot \omega_c)$$

$$P_{LR}(4 \cdot \omega_c) = P_{LR}(0) \quad P_{LR}(3 \cdot \omega_c) = P_{LR}(-\omega_c) \quad P_{LR}(5 \cdot \omega_c) = P_{LR}(\omega_c)$$

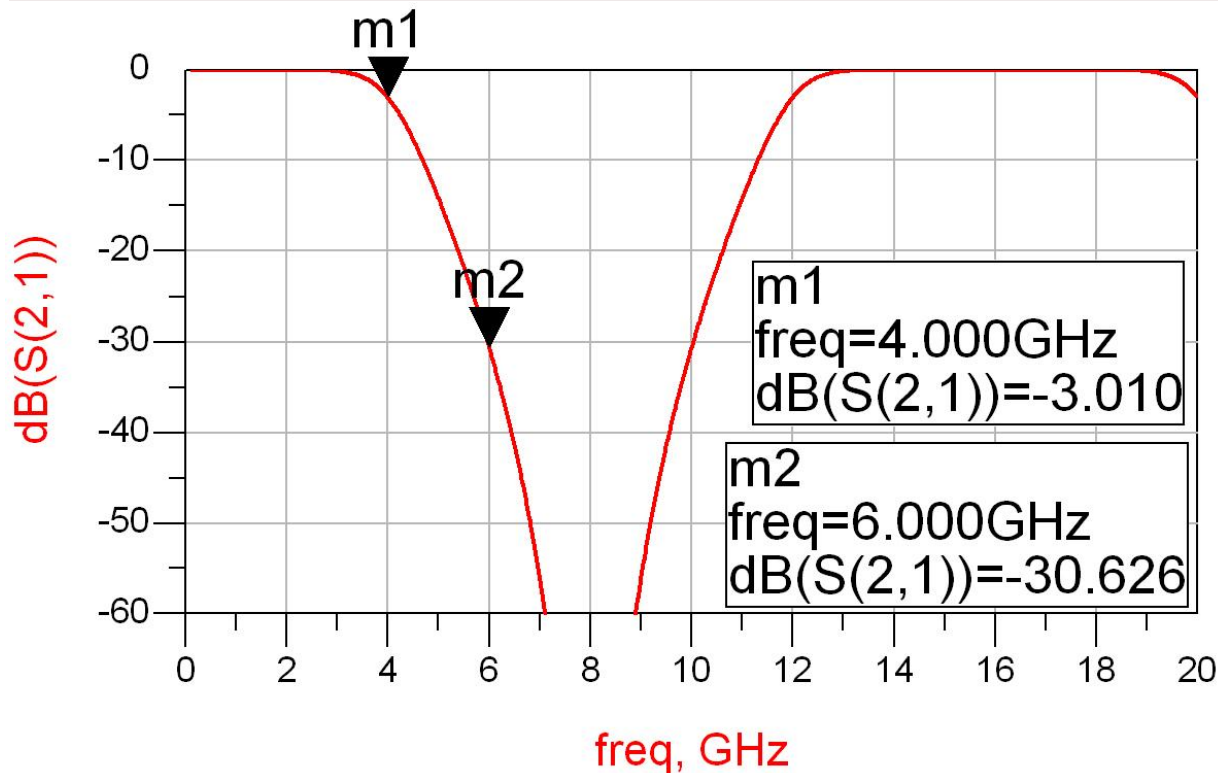
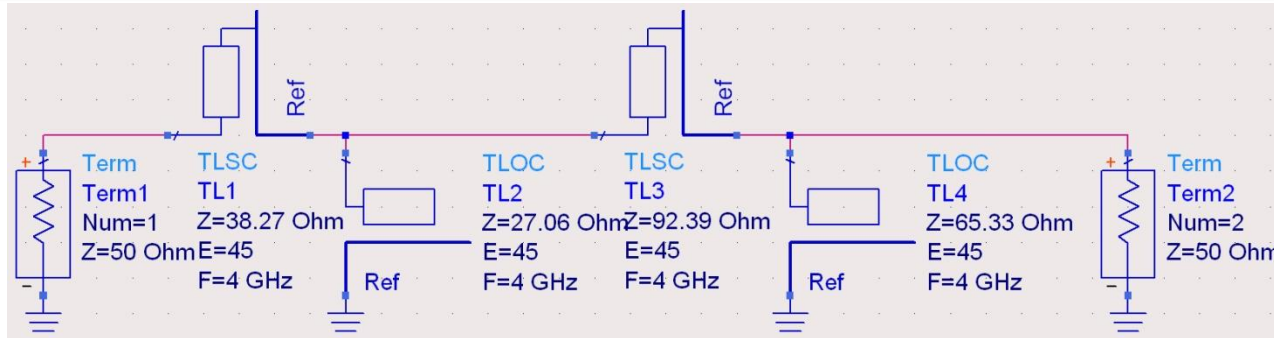
# Example

- Low-pass filter 4<sup>th</sup> order, 4 GHz cutoff frequency, maximally flat design (working with 50Ω source and load)
- maximally flat table or formulas:
  - $g_1 = 0.7654 = L_1$
  - $g_2 = 1.8478 = C_2$
  - $g_3 = 1.8478 = L_3$
  - $g_4 = 0.7654 = C_4$
  - $g_5 = 1$  (**does not** need supplemental impedance matching – required only for even order equal-ripple filters)

# Lumped elements – ADS

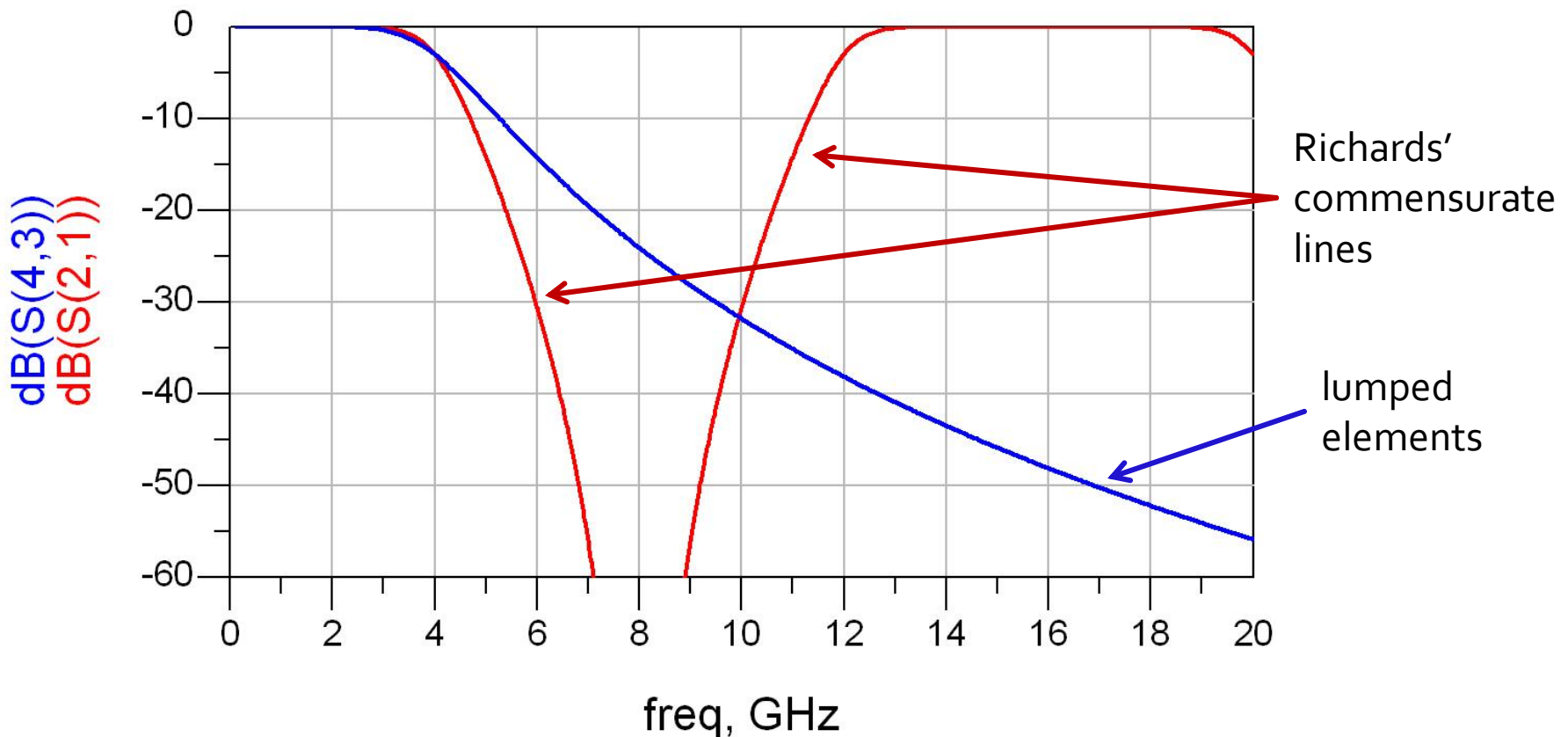


# Richards' Transformation – ADS



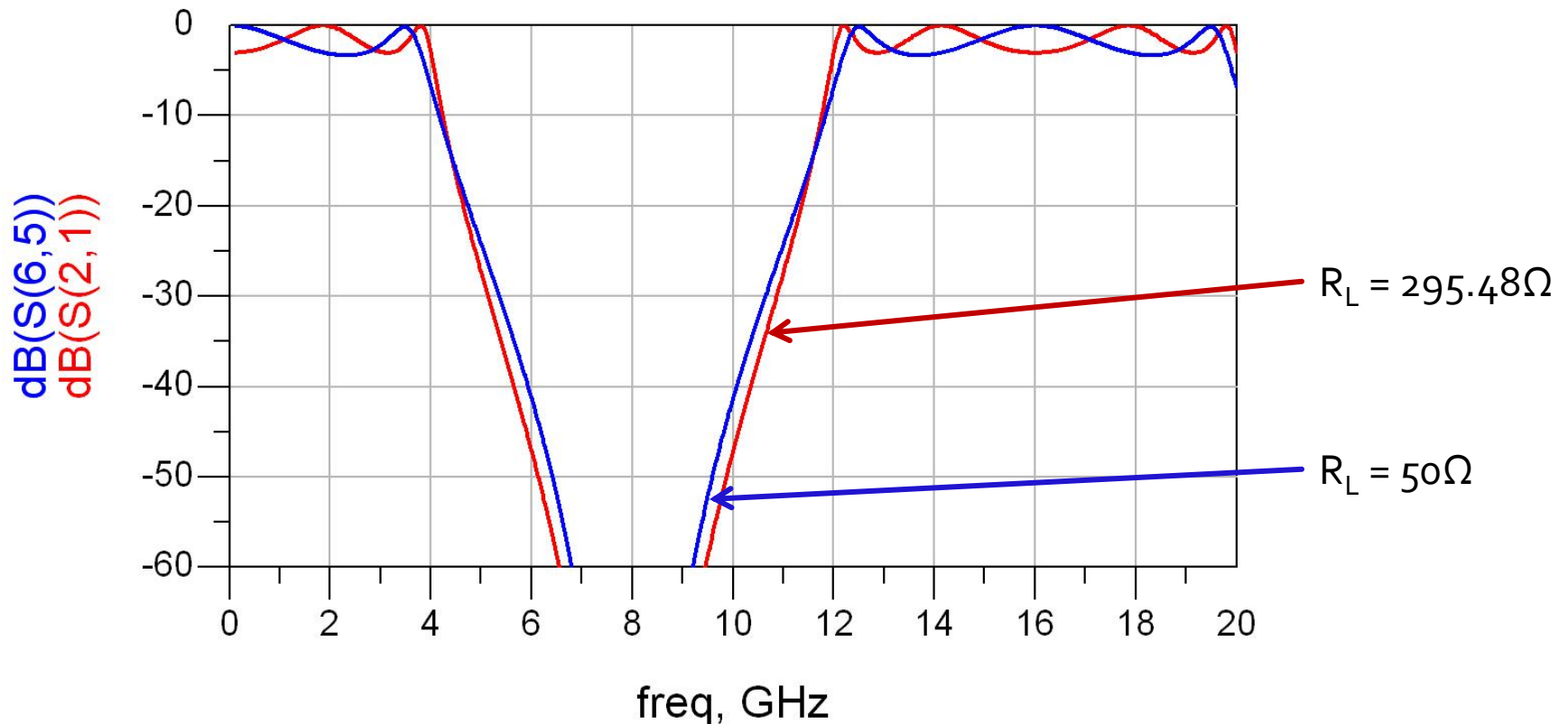
# Richards' Transformation

- Filters implemented with Richards' Transformation
  - beneficiate from the supplemental pole at  $2 \cdot \omega_c$
  - have the major disadvantage of frequency periodicity, a supplemental non-periodic LPF must be inserted if needed



# Observation: even order equal-ripple

- Even order equal-ripple filters need output matching towards  $50\Omega$  for precise results.  
Example:

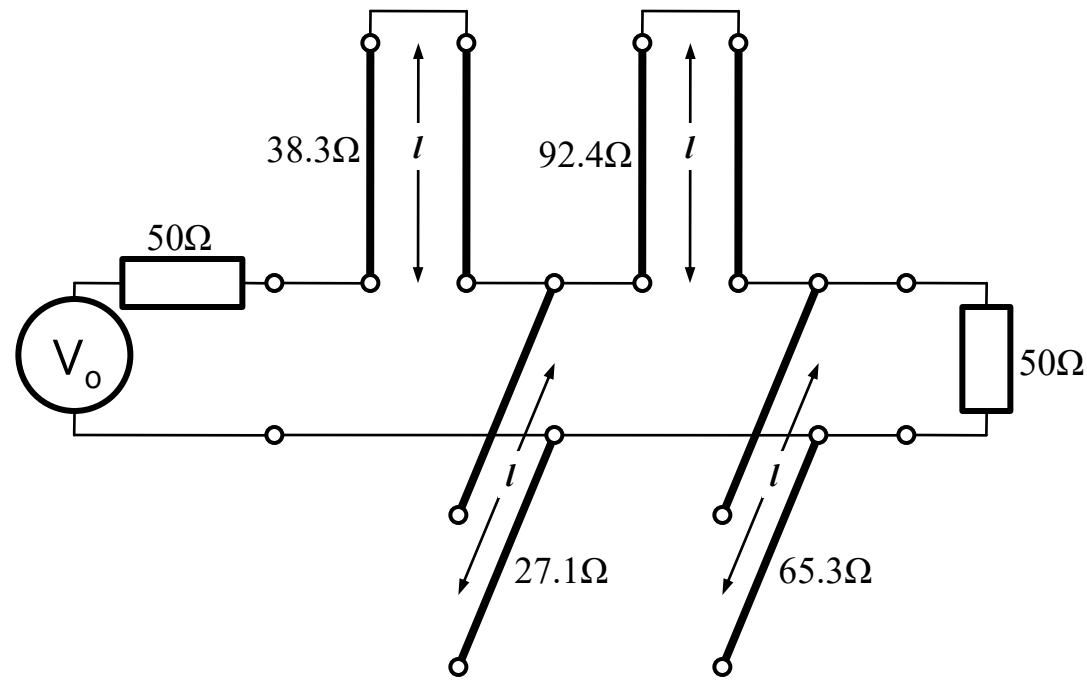


**Continue**

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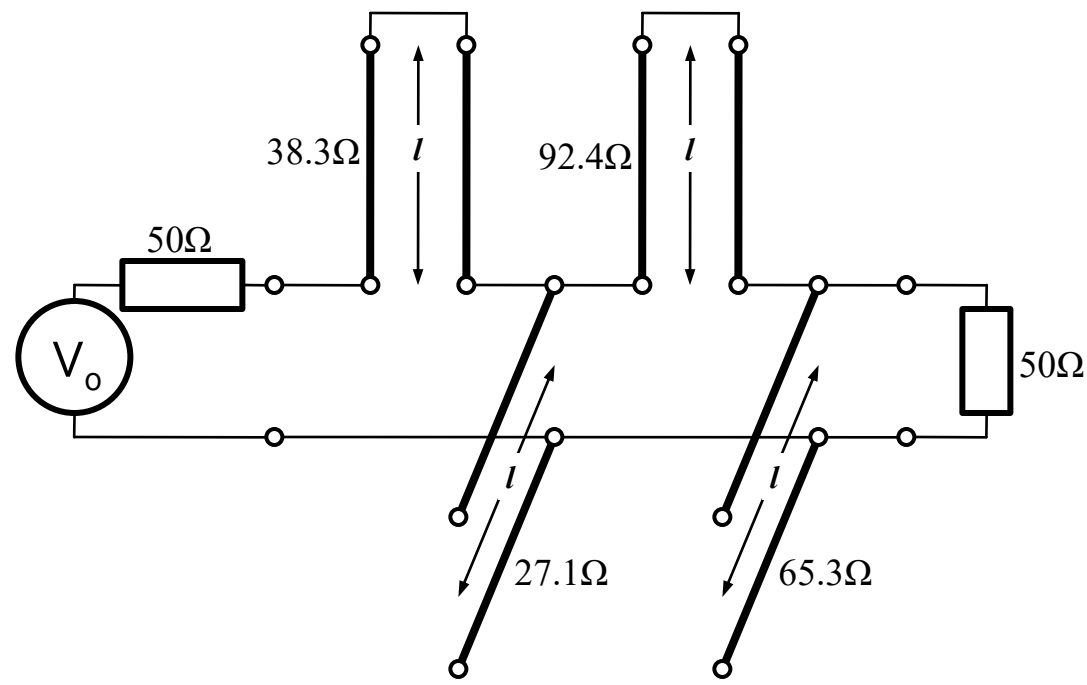
# Kuroda's Identities

- Filters implemented with the Richards' transformation have certain disadvantages in terms of practical use
- Kuroda's Identities/Transformations can eliminate some of these disadvantages
- We use additional line sections to obtain systems that are easier to implement in practice
- The additional line sections are called unit elements and have lengths of  $\lambda / 8$  at the desired cutoff frequency ( $\omega_c$ ) thus being commensurate with the stubs implementing the inductors and capacitors.



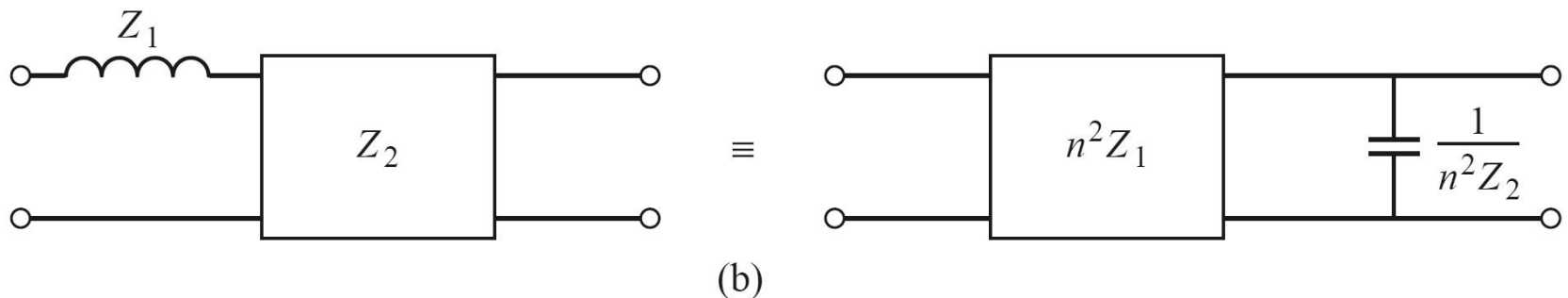
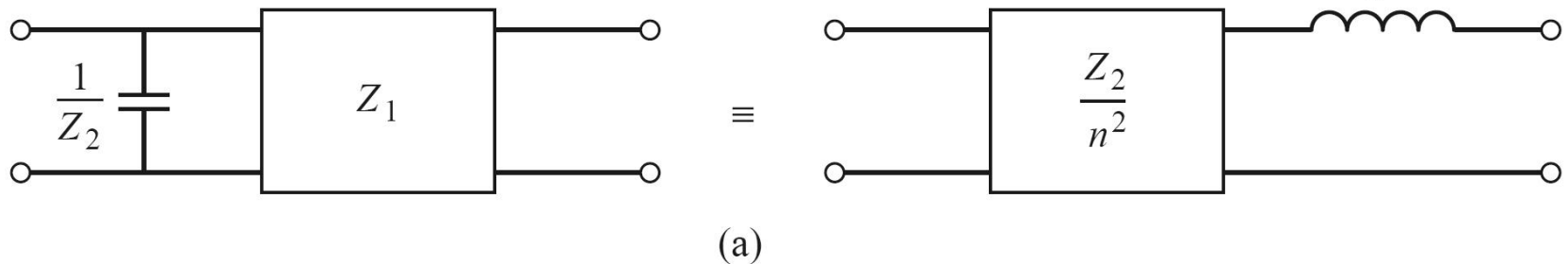
# Kuroda's Identities

- Kuroda's Identities perform any of the following operations:
  - Physically separate transmission line stubs
  - Transform series stubs into shunt stubs, or vice versa
  - Change impractical characteristic impedances into more realizable values ( $\sim 50\Omega$ )



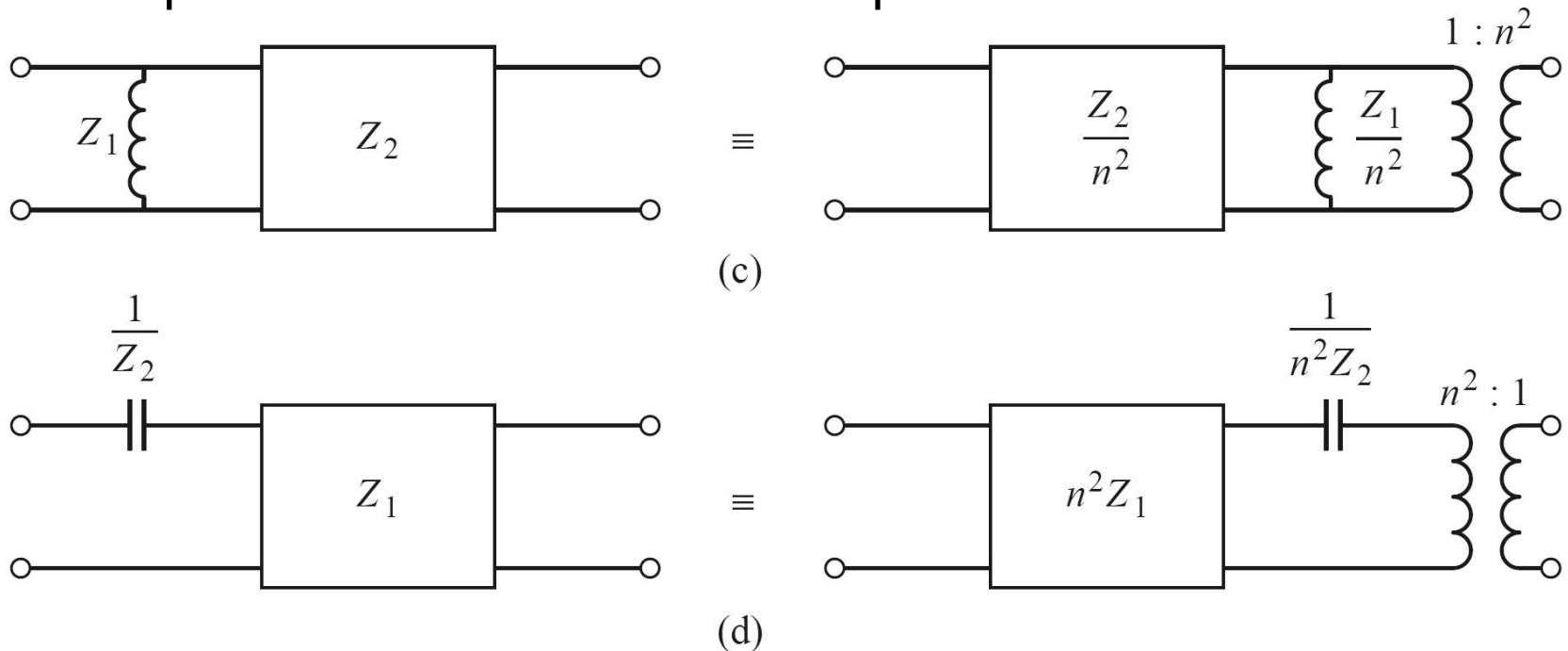
# Kuroda's Identities

- 4 circuit equivalents (a,b)
  - each box represents a unit element, or transmission line, of the indicated characteristic impedance and length ( $\lambda/8$  at  $\omega_c$ ). The inductors and capacitors represent short-circuit and open-circuit stubs  $\frac{Z_1}{n^2}$



# Kuroda's Identities

- 4 circuit equivalents (c,d)
  - each box represents a unit element, or transmission line, of the indicated characteristic impedance and length ( $\lambda/8$  at  $\omega_c$ ). The inductors and capacitors represent short-circuit and open-circuit stubs



# Kuroda's Identities

- In all Kuroda's Identities:

- n:

$$n^2 = 1 + \frac{Z_2}{Z_1}$$

- The inductors and capacitors represent short-circuit and open-circuit stubs resulted from Richards' transformation ( $\lambda/8$  at  $\omega_c$ ).
    - Each box represents a unit element, or transmission line, of the indicated characteristic impedance and length ( $\lambda/8$  at  $\omega_c$ ).

# First Kuroda's Identity

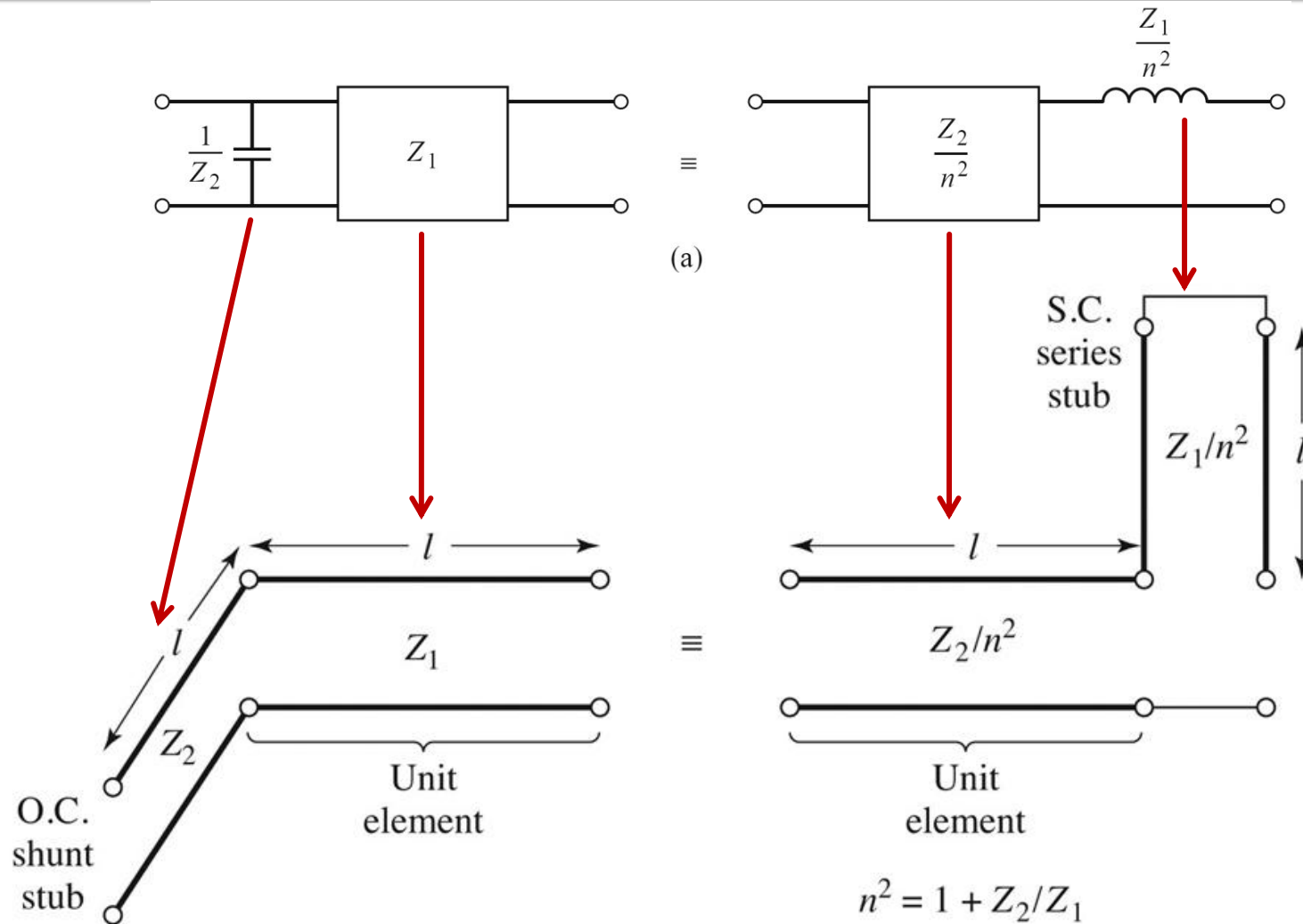
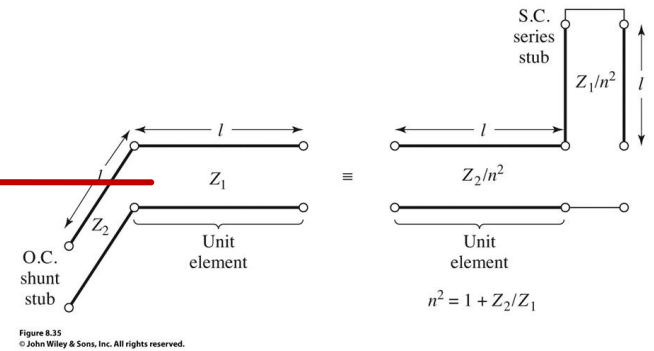
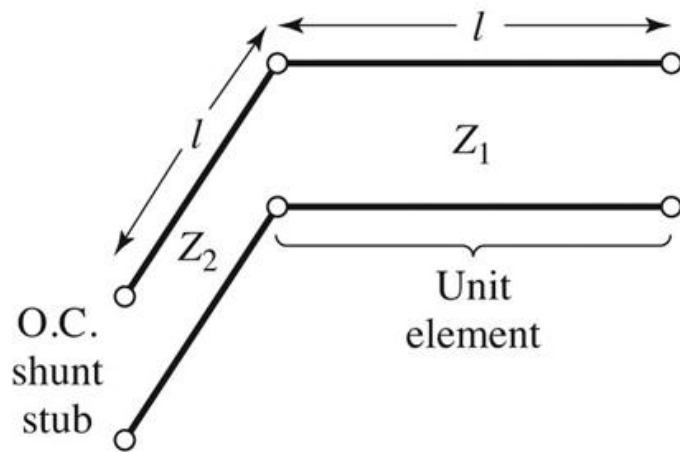
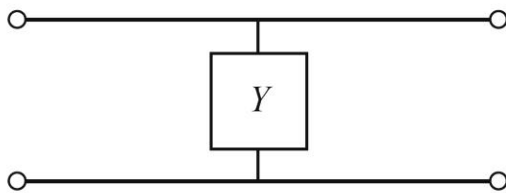


Figure 8.35  
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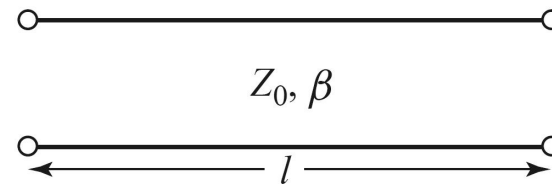
# First Kuroda's Identity – Proof



## ■ ABCD matrices, $L_4$



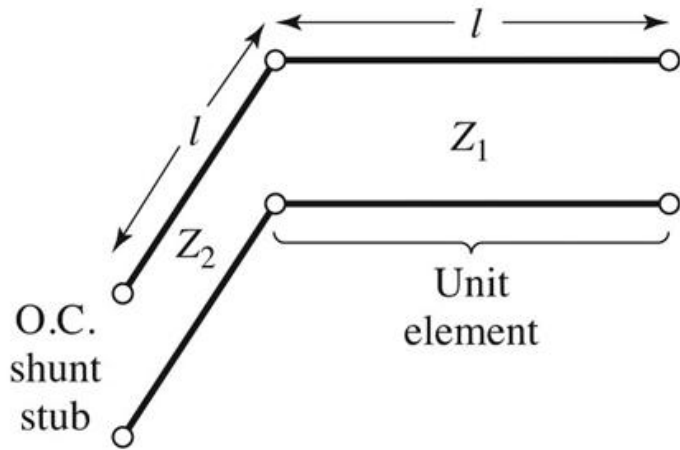
+



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos \beta \cdot l & j \cdot Z_0 \cdot \sin \beta \cdot l \\ j \cdot Y_0 \cdot \sin \beta \cdot l & \cos \beta \cdot l \end{bmatrix}$$

# First Kuroda's Identity – Proof



$$\Omega = \tan \beta \cdot l$$

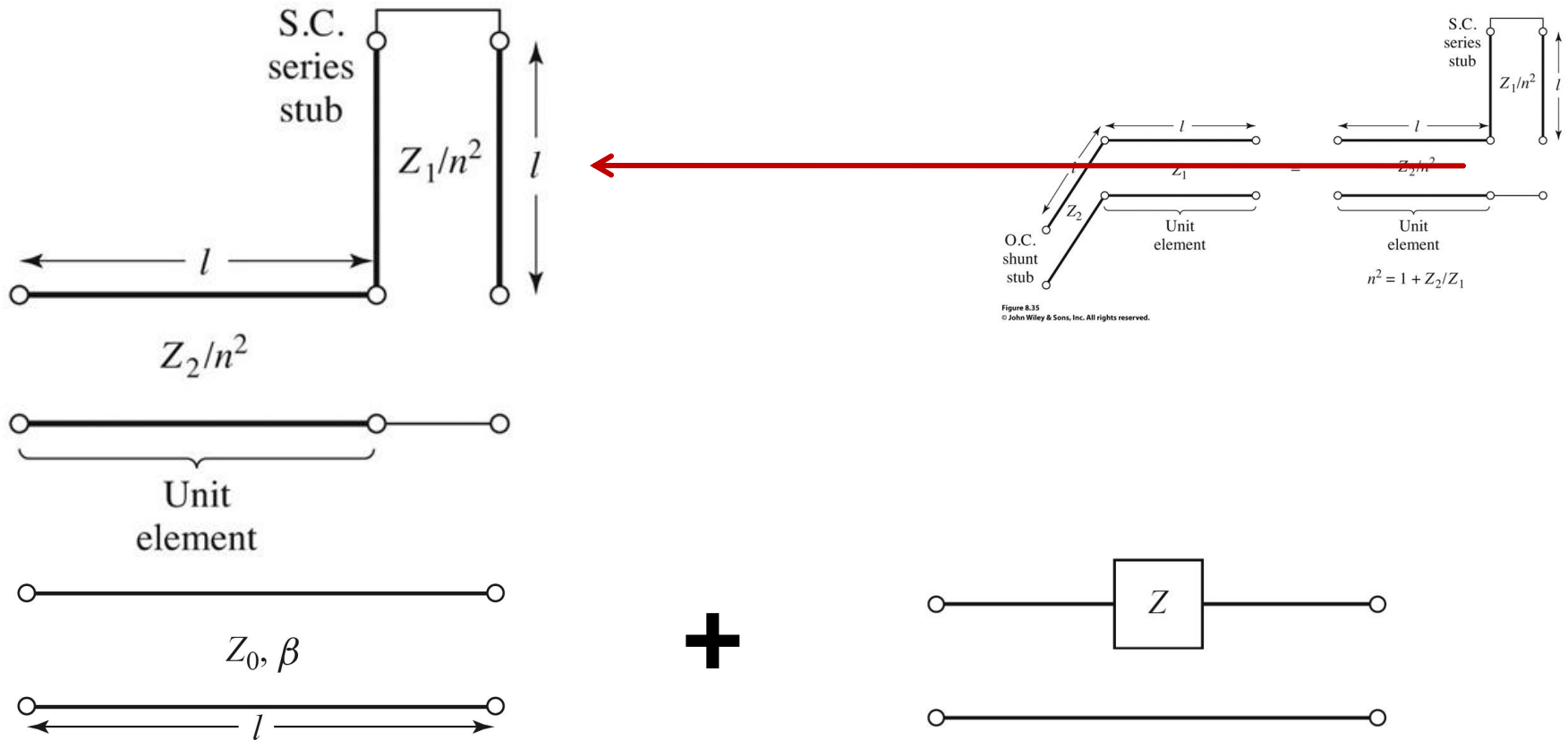
$$\cos \beta \cdot l = \frac{1}{\sqrt{1 + \Omega^2}} \quad \sin \beta \cdot l = \frac{\Omega}{\sqrt{1 + \Omega^2}}$$

$$Z_{in,oc} = -j \cdot Z_2 \cdot \cot \beta \cdot l = -j \cdot \frac{Z_2}{\Omega}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{j \cdot \Omega}{Z_2} & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{1 + \Omega^2}} & j \cdot Z_1 \cdot \frac{\Omega}{\sqrt{1 + \Omega^2}} \\ j \cdot \frac{1}{Z_1} \cdot \frac{\Omega}{\sqrt{1 + \Omega^2}} & \frac{1}{\sqrt{1 + \Omega^2}} \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{\sqrt{1 + \Omega^2}} \cdot \begin{bmatrix} 1 & 0 \\ \frac{j \cdot \Omega}{Z_2} & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & j \cdot \Omega \cdot Z_1 \\ \frac{j \cdot \Omega}{Z_1} & 1 \end{bmatrix} = \frac{1}{\sqrt{1 + \Omega^2}} \cdot \begin{bmatrix} 1 & j \cdot \Omega \cdot Z_1 \\ j \cdot \Omega \cdot \left( \frac{1}{Z_1} + \frac{1}{Z_2} \right) & 1 - \Omega^2 \cdot \frac{Z_1}{Z_2} \end{bmatrix}$$

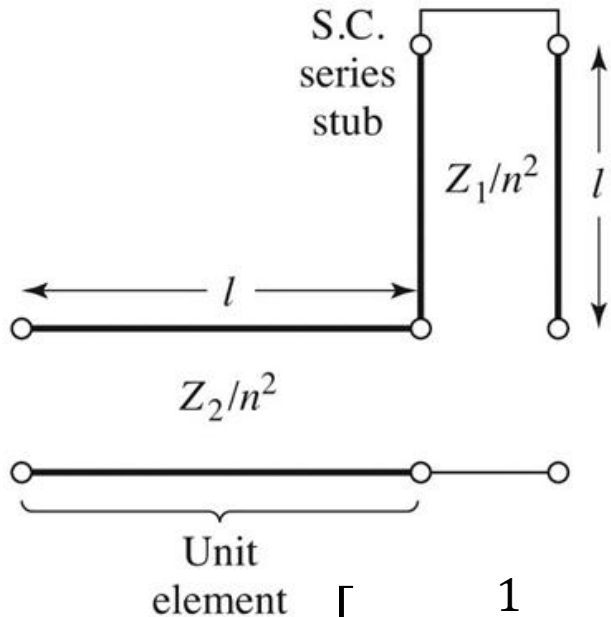
# First Kuroda's Identity – Proof



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos \beta \cdot l & j \cdot Z_0 \cdot \sin \beta \cdot l \\ j \cdot Y_0 \cdot \sin \beta \cdot l & \cos \beta \cdot l \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

# First Kuroda's Identity – Proof



$$\Omega = \tan \beta \cdot l$$

$$\cos \beta \cdot l = \frac{1}{\sqrt{1 + \Omega^2}} \quad \sin \beta \cdot l = \frac{\Omega}{\sqrt{1 + \Omega^2}}$$

$$Z_{in,sc} = j \cdot \left( \frac{Z_1}{n^2} \right) \cdot \tan \beta \cdot l = \frac{j \cdot \Omega \cdot Z_1}{n^2}$$

$$[A \quad B] = \begin{bmatrix} 1 & \\ \frac{1}{\sqrt{1 + \Omega^2}} & \\ j \cdot \frac{Z_2}{n^2} \cdot \frac{\Omega}{\sqrt{1 + \Omega^2}} & \\ \Omega & \end{bmatrix} \cdot \begin{bmatrix} j \cdot \frac{Z_2}{n^2} \cdot \frac{\Omega}{\sqrt{1 + \Omega^2}} & \\ 1 & \\ \frac{1}{\sqrt{1 + \Omega^2}} & \end{bmatrix} \cdot \begin{bmatrix} 1 & \frac{j \cdot \Omega \cdot Z_1}{n^2} \\ 0 & 1 \end{bmatrix}$$

$$[A \quad B] = \frac{1}{\sqrt{1 + \Omega^2}} \cdot \begin{bmatrix} 1 & j \cdot \Omega \cdot \frac{Z_2}{n^2} \\ j \cdot \Omega \cdot n^2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & j \cdot \Omega \cdot \frac{Z_1}{n^2} \\ 0 & 1 \end{bmatrix} = \frac{1}{\sqrt{1 + \Omega^2}} \cdot \begin{bmatrix} 1 & j \cdot \frac{\Omega}{n^2} \cdot (Z_1 + Z_2) \\ j \cdot \Omega \cdot n^2 & 1 - \Omega^2 \cdot \frac{Z_1}{Z_2} \end{bmatrix}$$

# First Kuroda's Identity – Proof

- First circuit

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{\sqrt{1 + \Omega^2}} \cdot \begin{bmatrix} 1 & j \cdot \Omega \cdot Z_1 \\ j \cdot \Omega \cdot \left( \frac{1}{Z_1} + \frac{1}{Z_2} \right) & 1 - \Omega^2 \cdot \frac{Z_1}{Z_2} \end{bmatrix}$$

- Second circuit

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{\sqrt{1 + \Omega^2}} \cdot \begin{bmatrix} 1 & j \cdot \frac{\Omega}{n^2} \cdot (Z_1 + Z_2) \\ \frac{j \cdot \Omega \cdot n^2}{Z_2} & 1 - \Omega^2 \cdot \frac{Z_1}{Z_2} \end{bmatrix}$$

- Results are identical if we choose

$$n^2 = 1 + \frac{Z_2}{Z_1}$$

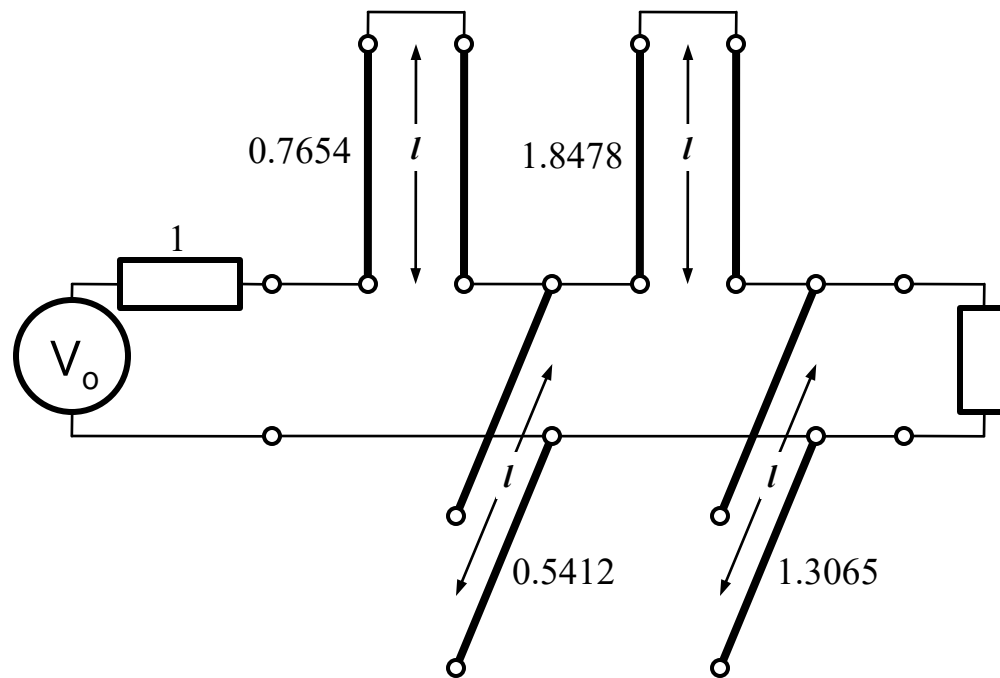
- The other 3 identities can be proved in the same way

# (Same) Example

- Low-pass filter 4<sup>th</sup> order, 4 GHz cutoff frequency, maximally flat design (working with 50Ω source and load)
- maximally flat table or formulas:
  - $g_1 = 0.7654 = L_1$
  - $g_2 = 1.8478 = C_2$
  - $g_3 = 1.8478 = L_3$
  - $g_4 = 0.7654 = C_4$
  - $g_5 = 1$  (**does not** need supplemental impedance matching – required only for even order equal-ripple filters)

# Example

- Apply Richards's transformation

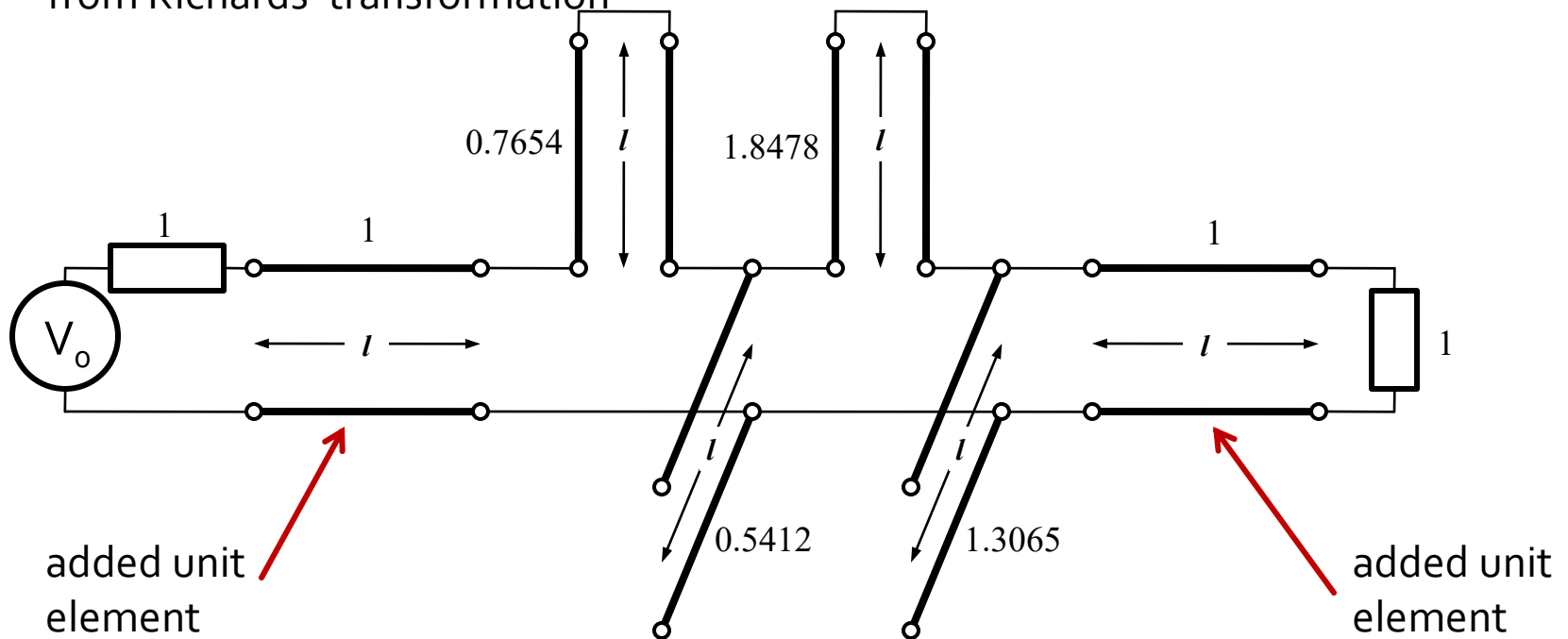


- Problems:

- the series stubs would be **very difficult** to implement in microstrip line technology
- in microstrip technology it is preferable to have open-circuit stubs (short-circuit requires a **via-hole** to the ground plane)
- 1 ■ the 4 stubs are physically connected at the same point, an implementation that eliminates/reduces the **coupling** between these lines is impossible
- not the case here, but sometimes the normalized impedances are much different from 1. Most circuit technologies are designed for  $50\Omega$  lines

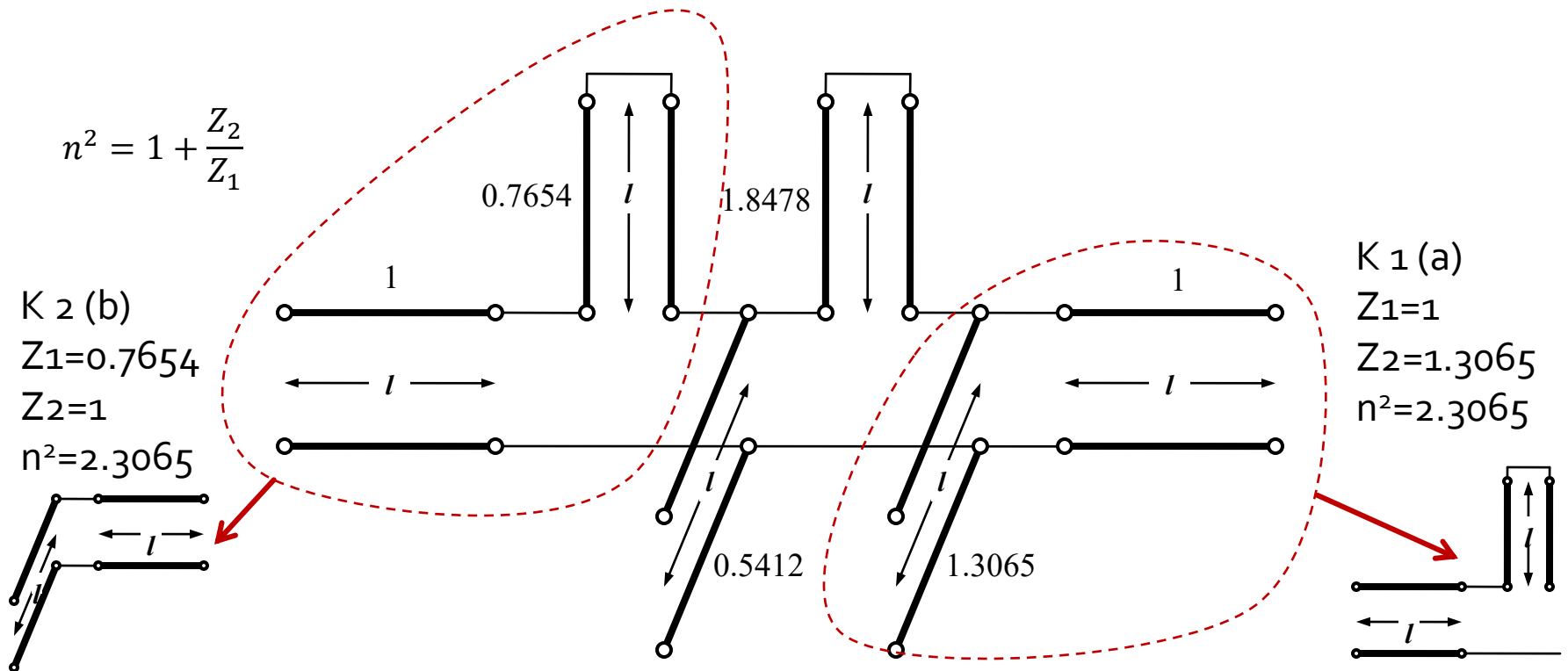
# Example

- In all 4 Kuroda's Identities we **always** have a circuit with a series line section (not present in initial circuit):
  - we **add** unit elements ( $z = 1, l = \lambda/8$ ) at the ends of the filter (these redundant elements do not affect filter performance since they are matched to  $z = 1$ , both source and load)
  - we **apply** one of the Kuroda's Identities at both ends and **continue** (add unit ...)
  - we can **stop** the procedure when we have a series line section between all the stubs from Richards' transformation



# Example

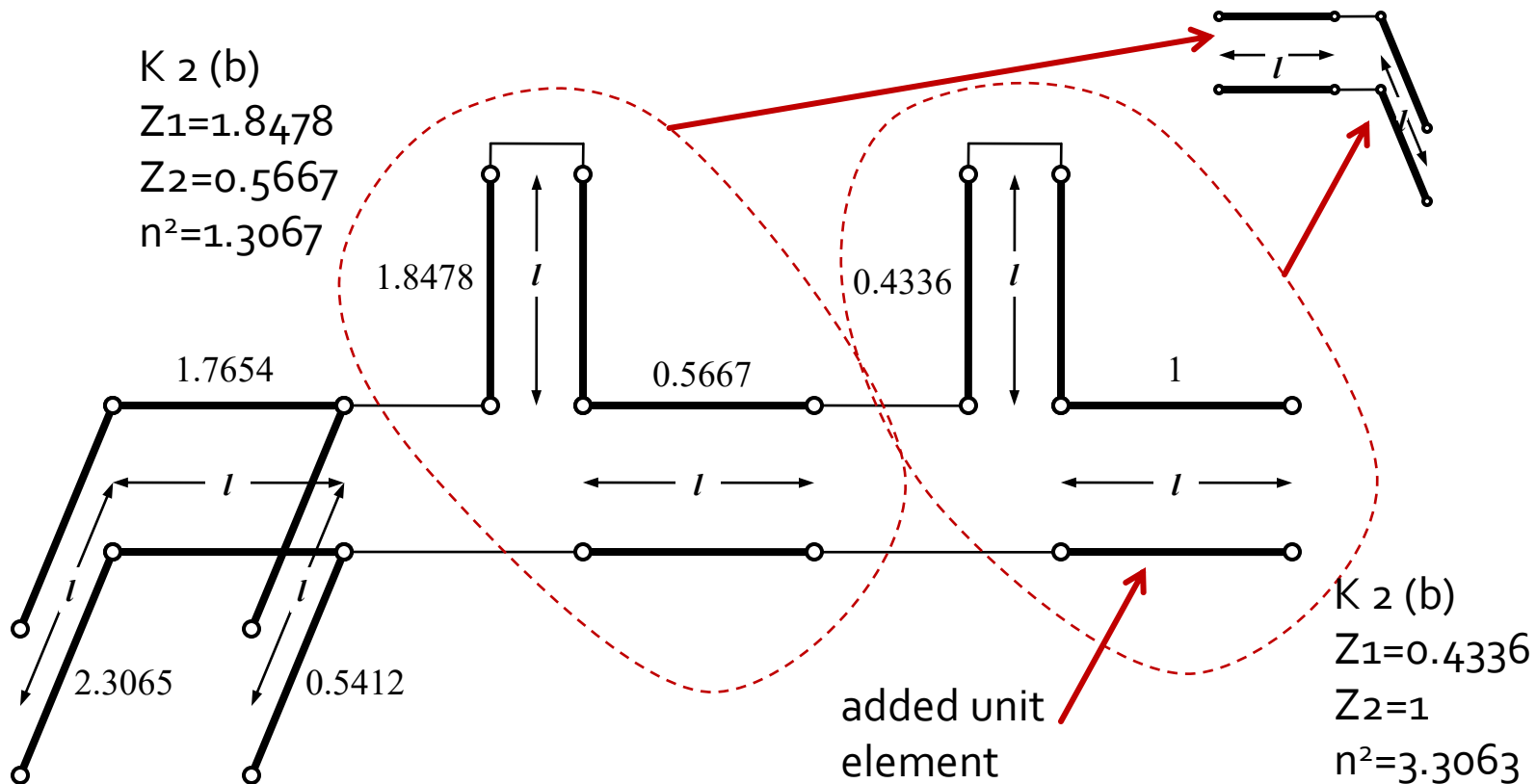
- Apply:
  - Kuroda 2 ( $L, Z$  known  $\rightarrow C, Z$ ) on the left side
  - Kuroda 1 ( $C, Z$  known  $\rightarrow L, Z$ ) on the right side



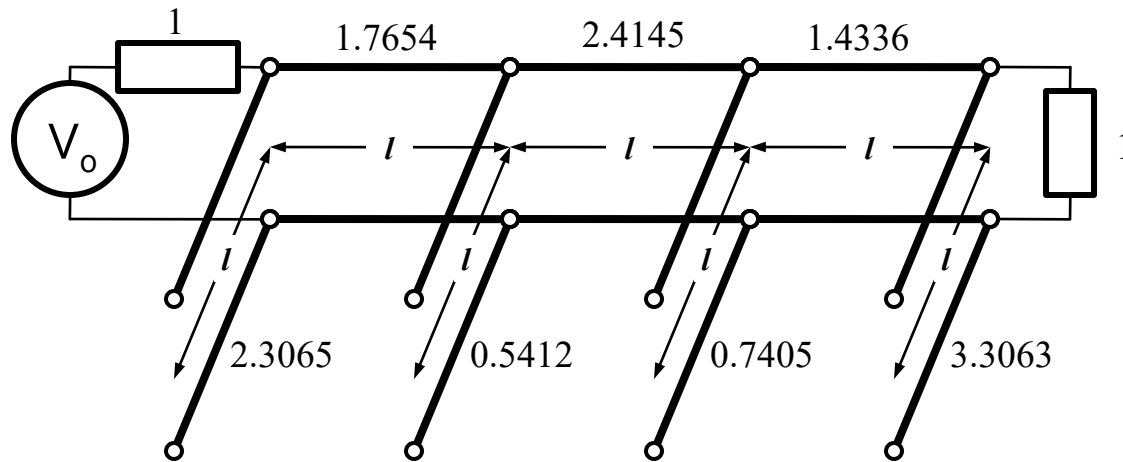
# Example

- We add another unit element on the right side and apply Kuroda 2 twice

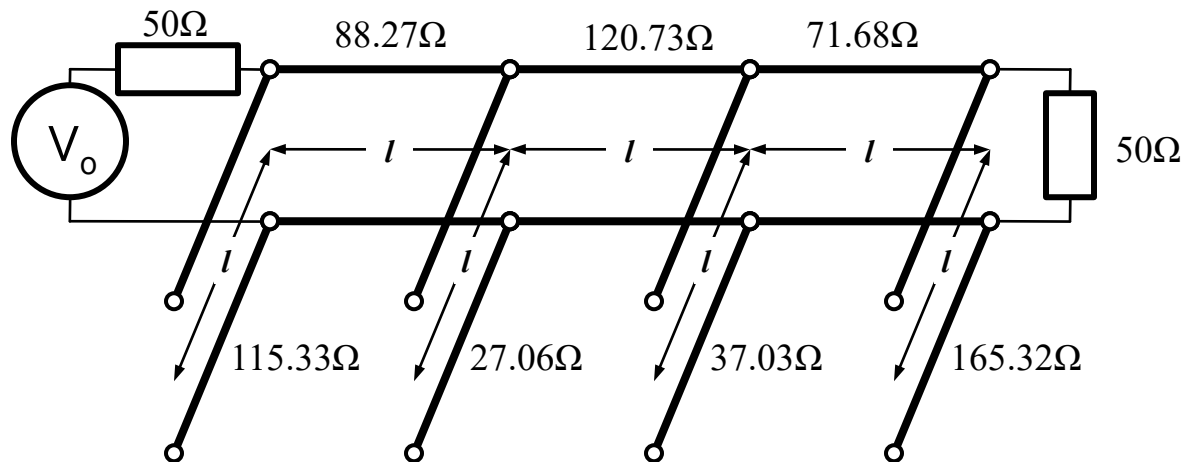
$$n^2 = 1 + \frac{Z_2}{Z_1}$$



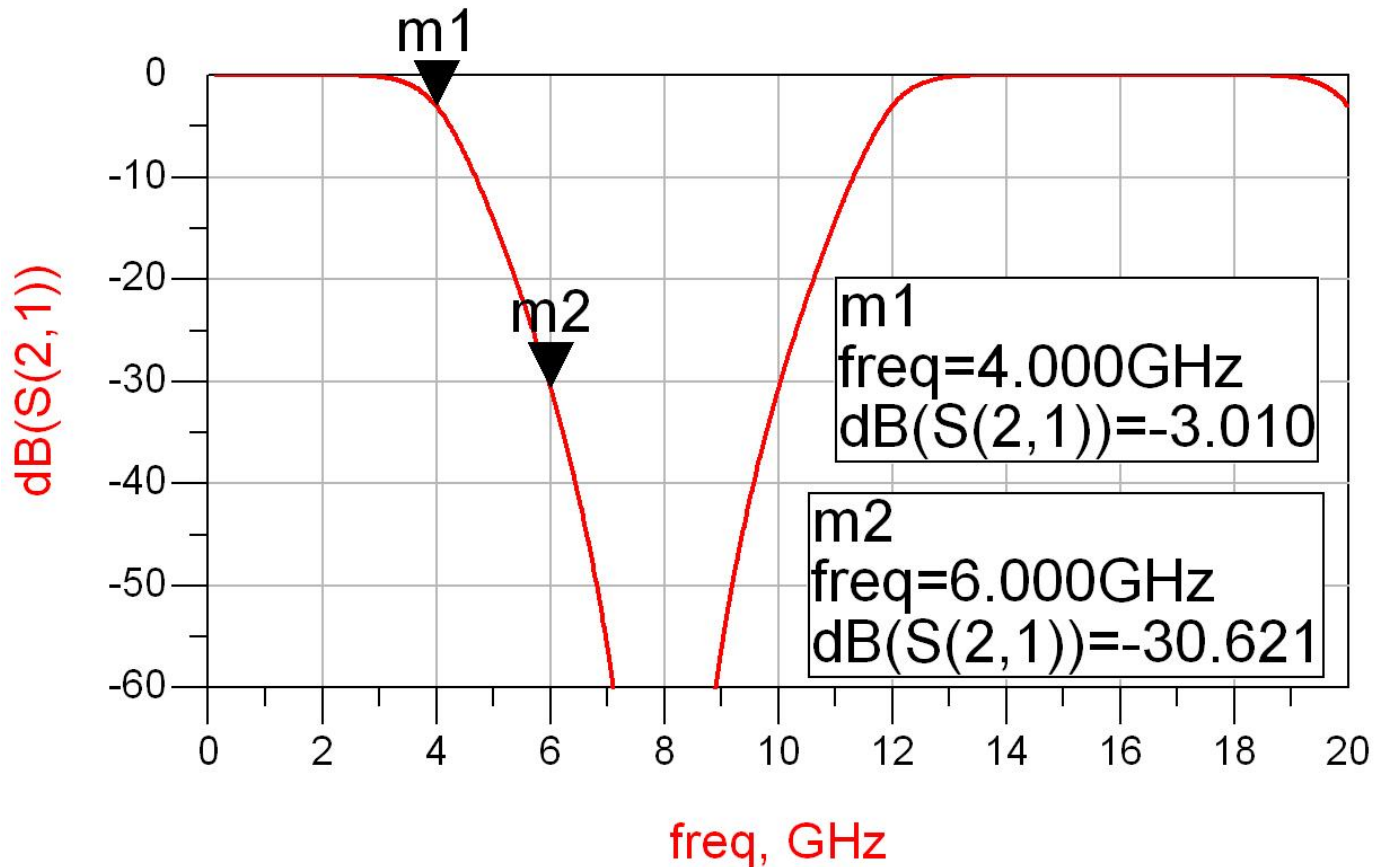
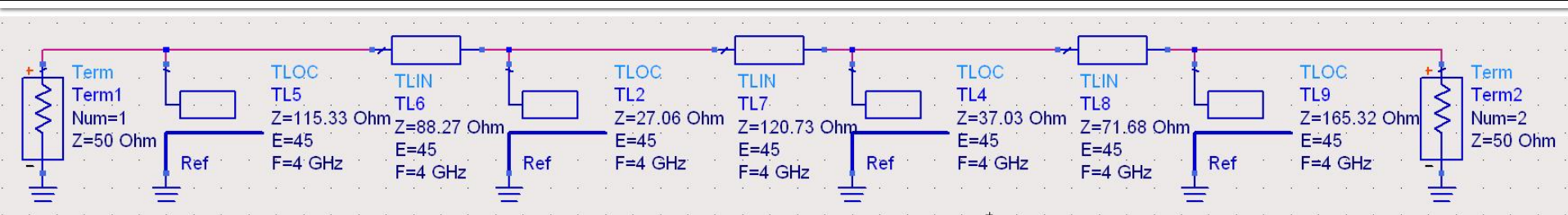
# Example



- Impedance scaling (multiply by  $50\Omega$ )



# Kuroda's Identities – ADS



# Examples

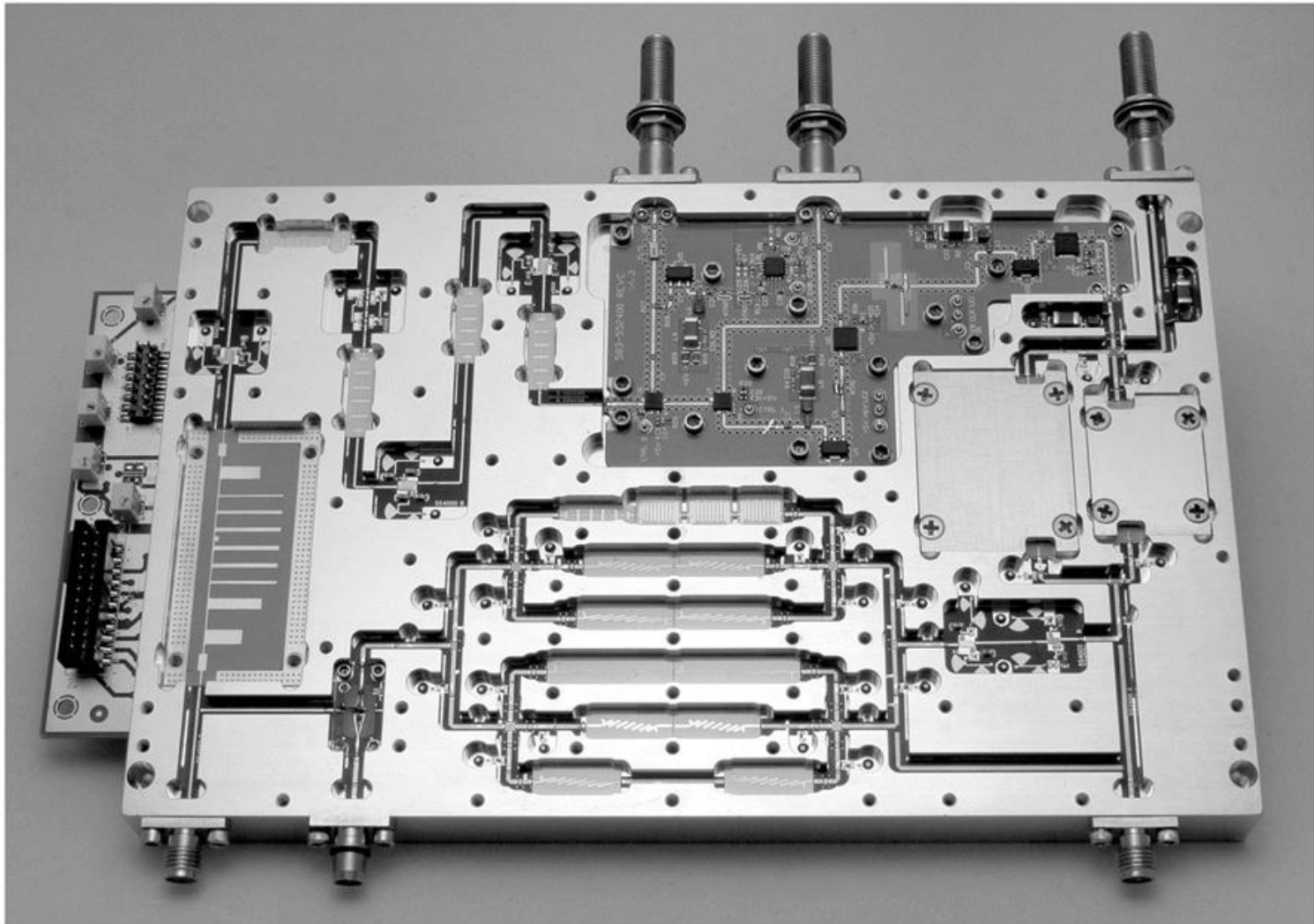


Figure 8.55  
Courtesy of LNX Corporation, Salem, N.H.

# Examples

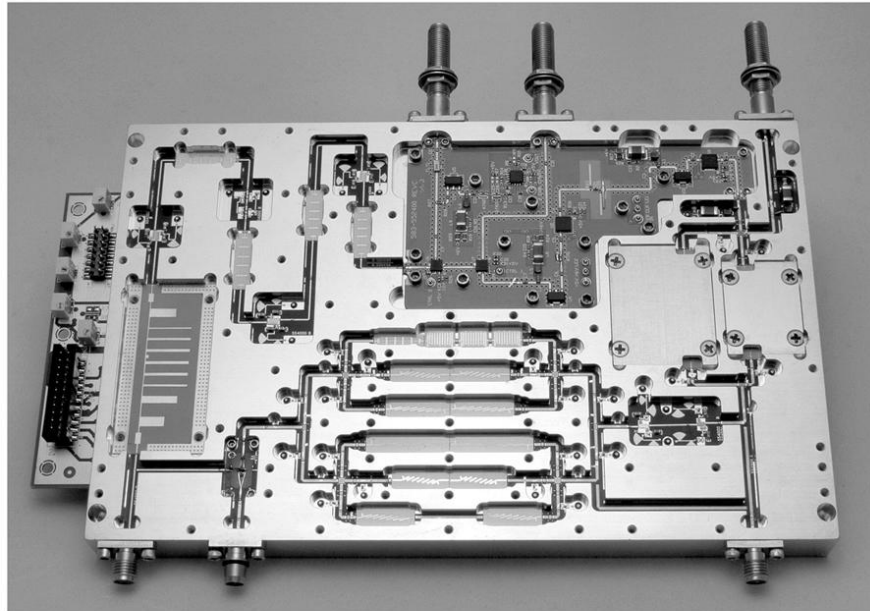
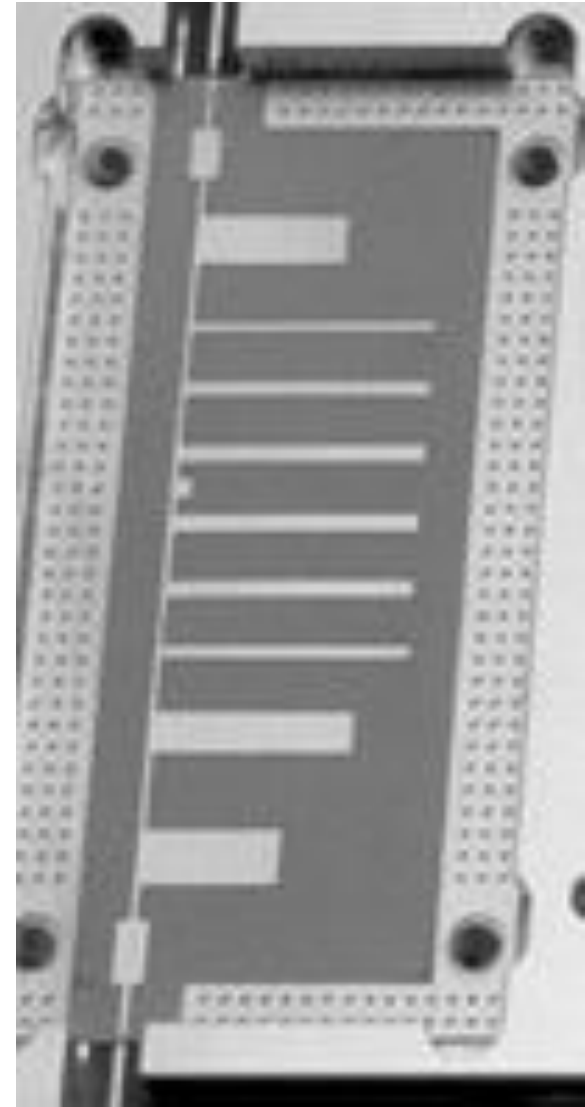
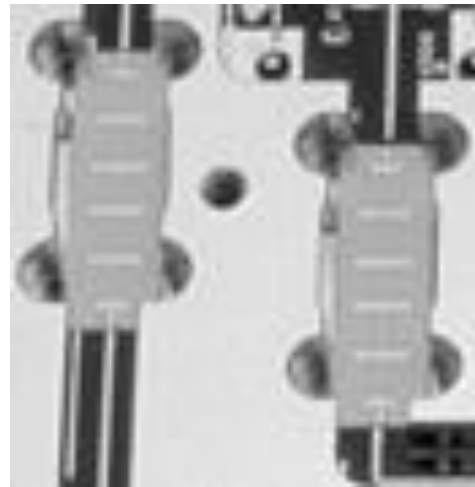
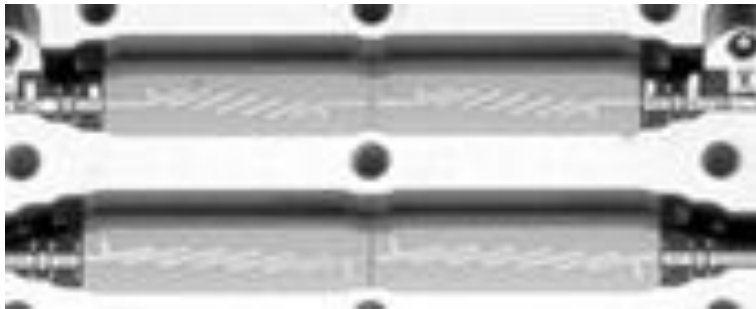


Figure 8.55  
Courtesy of LNX Corporation, Salem, N.H.



# Impedance and Admittance Inverters

- Richards' transformation and Kuroda's identities are useful especially for low-pass filters in technologies where the series stubs would be very difficult/impossible to implement (microstrip)
- In the case of other filters (example 3<sup>rd</sup> order BPF):
  - series inductance can be implemented using K1-K2
  - series capacitance cannot be implemented using shunt stubs

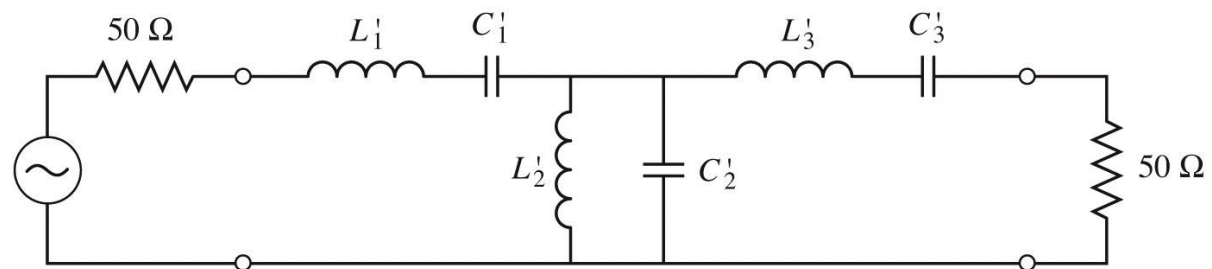


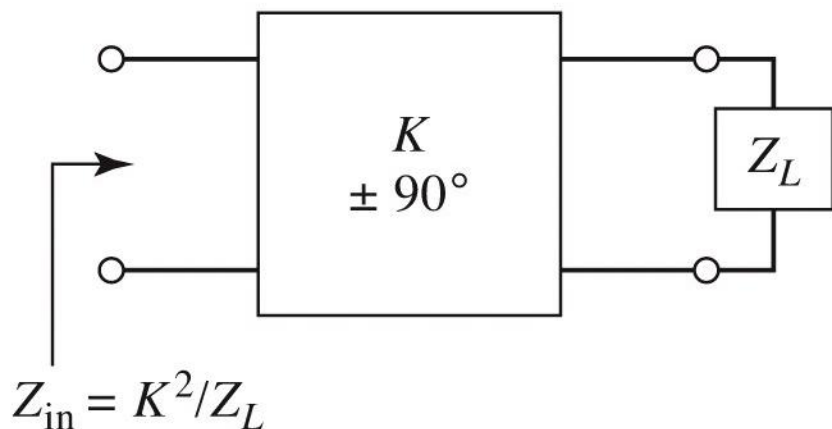
Figure 8.32  
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# Impedance and Admittance Inverters

- For cases where Richards + Kuroda do not offer practical solutions we use circuits called **impedance and admittance inverters**

$$Z_{in} = \frac{K^2}{Z_L}$$

Impedance inverters



$$Y_{in} = \frac{J^2}{Y_L}$$

Admittance inverters

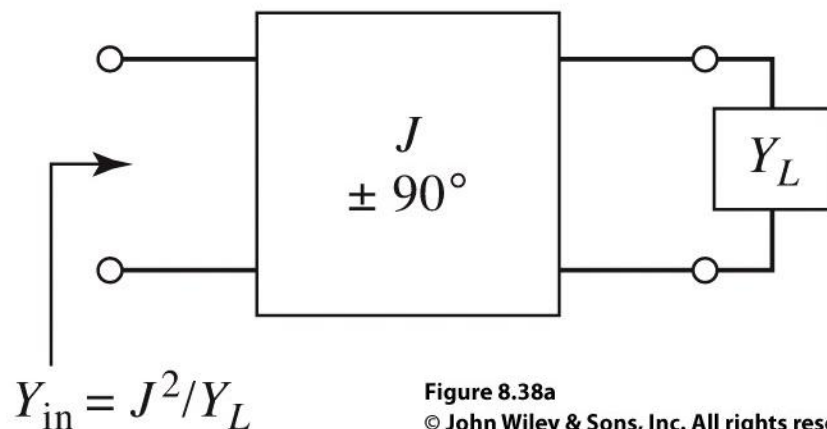


Figure 8.38a

# Impedance and Admittance Inverters

- The simplest example of impedance and admittance inverter is the **quarter-wave transformer** (L3)

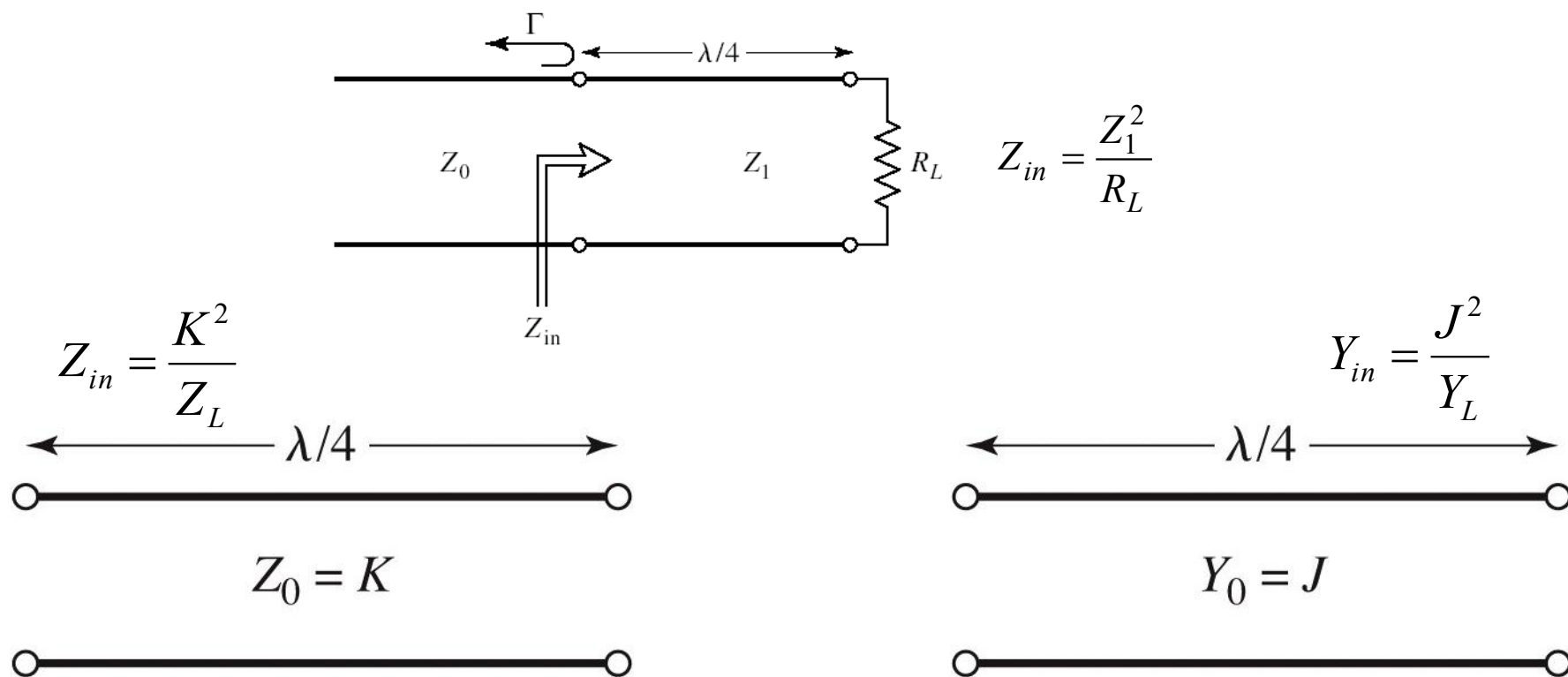
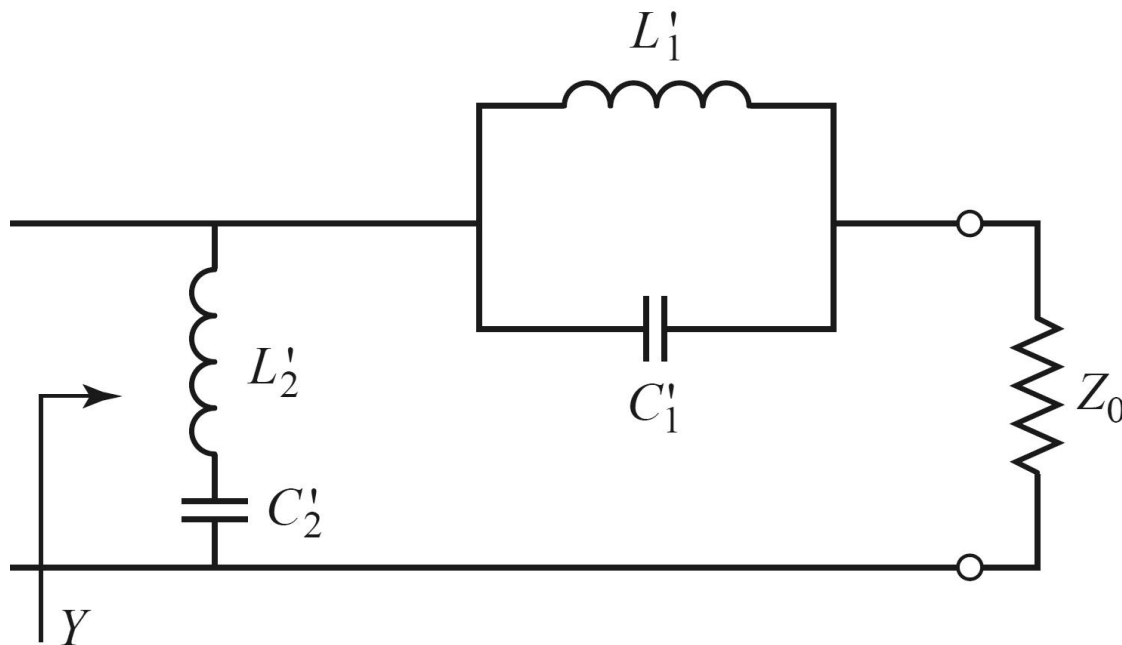


Figure 8.38b

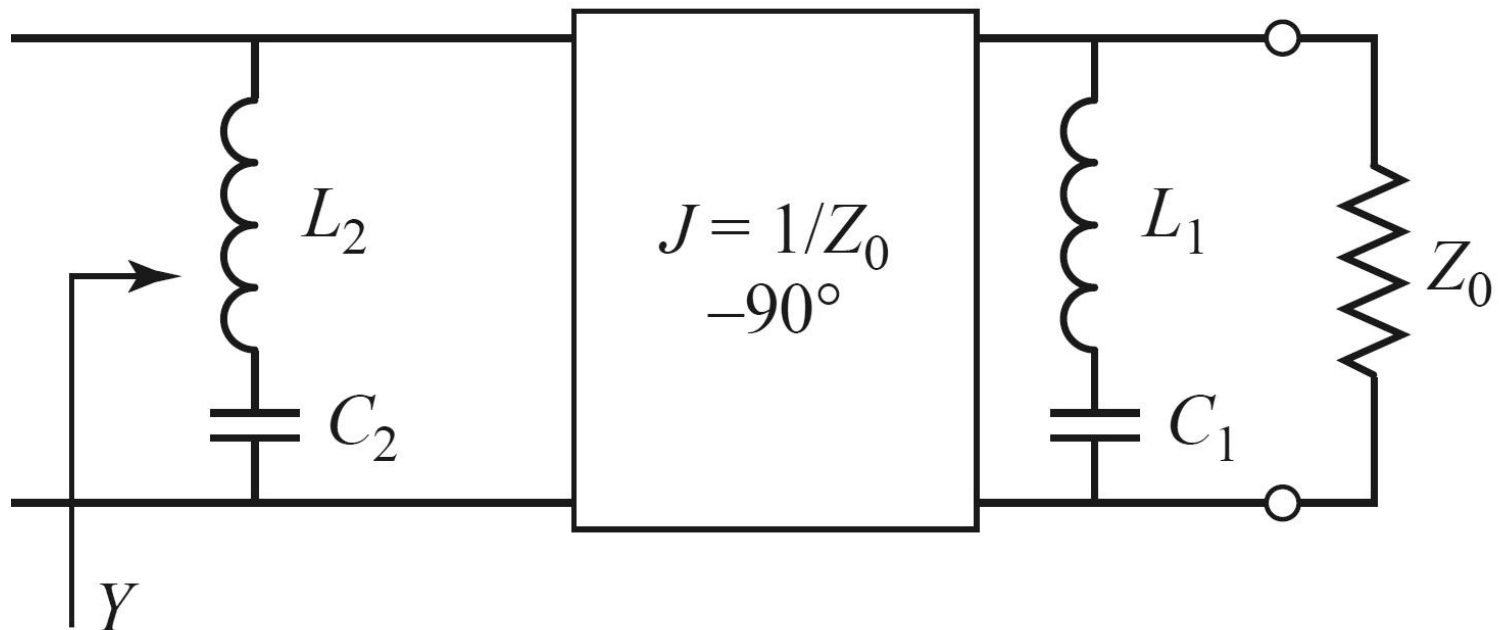
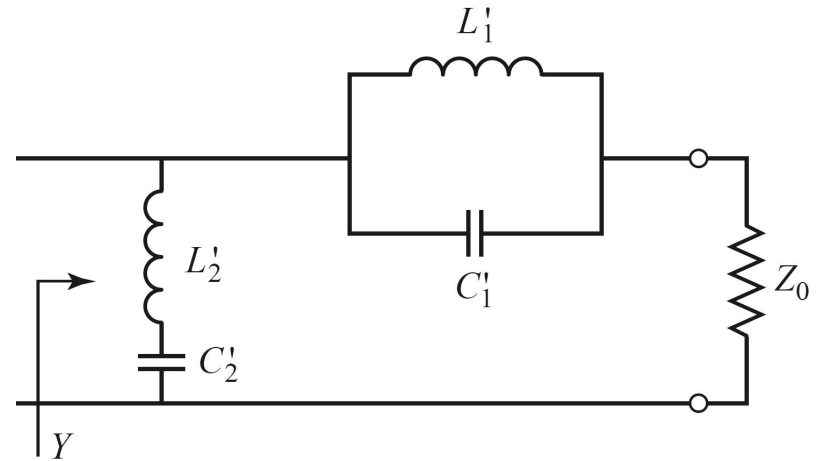
# Impedance and Admittance Inverters

- Impedance/admittance inverters can be used to change the structure of a designed filter to a realizable form
- For example a 2<sup>nd</sup> order BSF



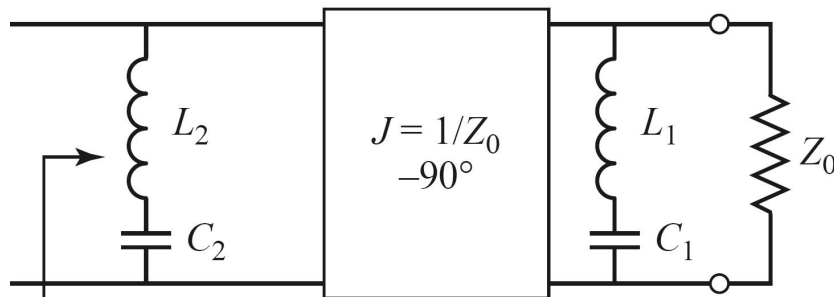
# Impedance and Admittance Inverters

- The series elements can be eliminated/replaced using an admittance inverter



# Impedance and Admittance Inverters

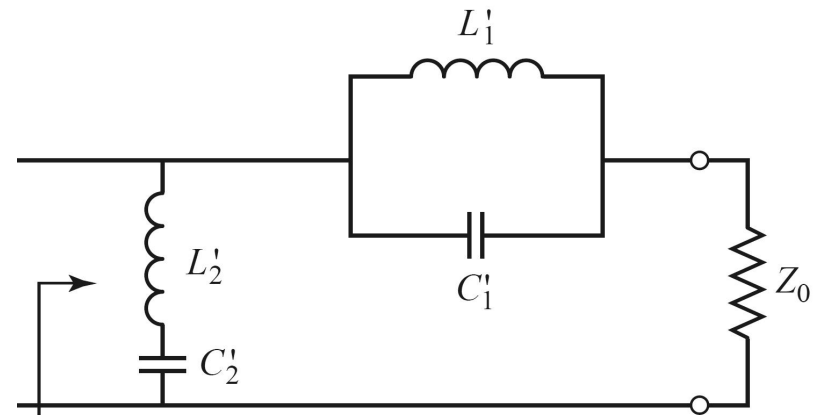
- The equivalence of the two schematics (when looking from the left) is proved by obtaining the same input admittance



$$Y = \frac{1}{j \cdot \omega \cdot L_2 + \frac{1}{j \cdot \omega \cdot C_2}} + \frac{1}{Z_0} \cdot \left( \frac{1}{j \cdot \omega \cdot L_1 + \frac{1}{j \cdot \omega \cdot C_1}} + \frac{1}{Z_0} \right)^{-1}$$

$$L_n \cdot C_n = L'_n \cdot C'_n = \frac{1}{\omega_0^2} \Rightarrow \frac{1}{Z_0^2} \cdot \sqrt{\frac{L_1}{C_1}} = \sqrt{\frac{C'_1}{L'_1}} \Rightarrow Y = Y'$$

$$\sqrt{\frac{L_2}{C_2}} = \sqrt{\frac{L'_2}{C'_2}}$$

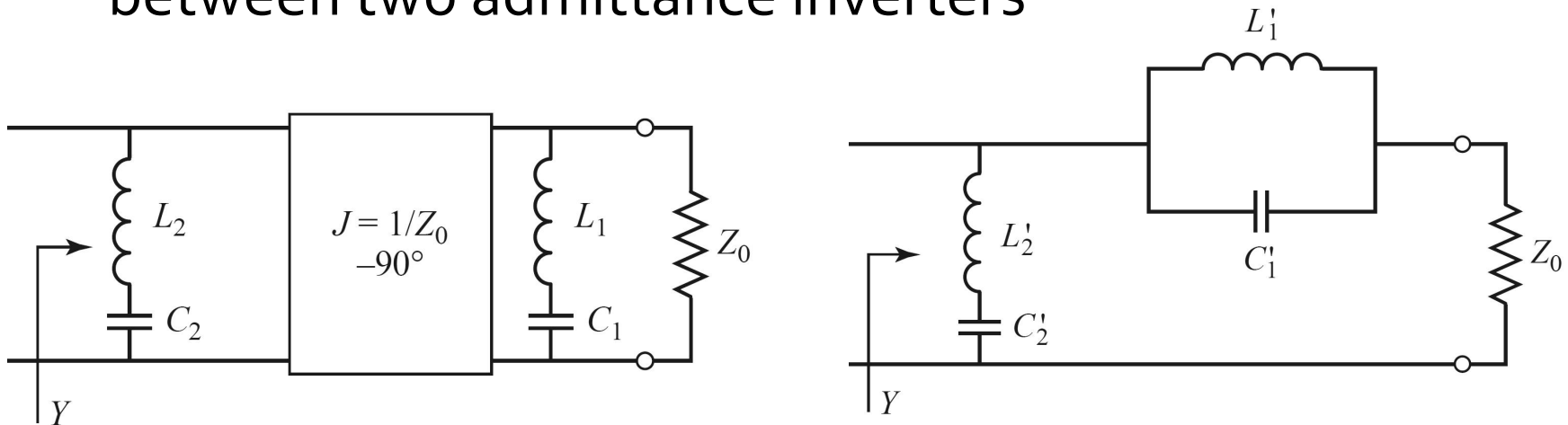


$$Y' = \frac{1}{j \cdot \omega \cdot L'_2 + \frac{1}{j \cdot \omega \cdot C'_2}} + \left( \frac{1}{j \cdot \omega \cdot C'_1 + \frac{1}{j \cdot \omega \cdot L'_1}} + Z_0 \right)^{-1}$$

- A similar result can be obtained for a bandpass filter

# Impedance and Admittance Inverters

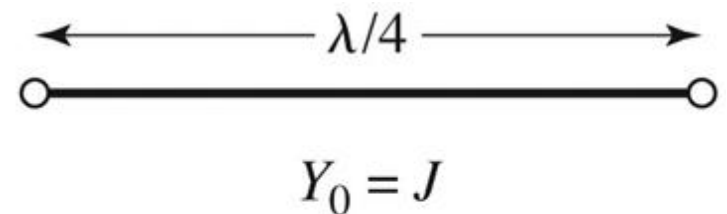
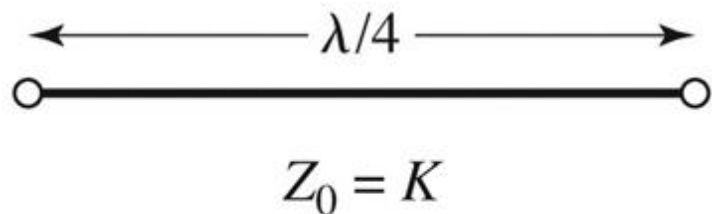
- The complete equivalence (when looking from both sides) is obtained by enclosing the series LC circuit between two admittance inverters



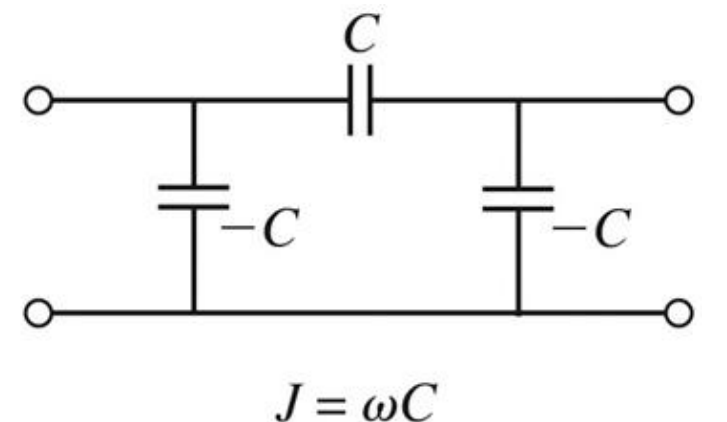
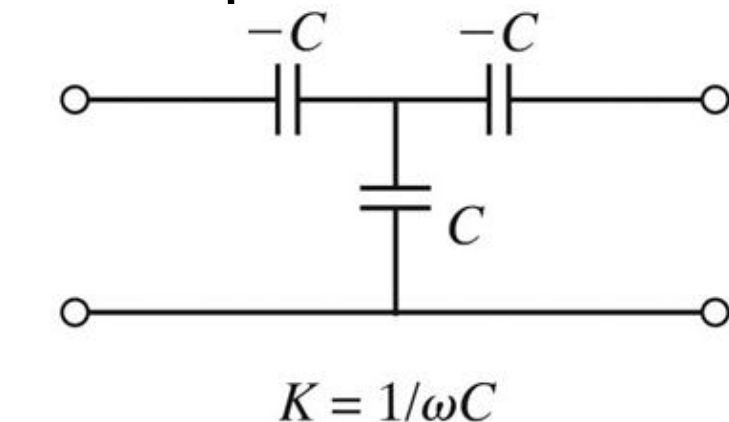
- A **series LC** circuit inserted in series in the circuit can be replaced by a **shunt LC** circuit inserted in parallel enclosed between 2 admittance inverters
- A **shunt LC** circuit inserted in series in the circuit can be replaced by a **series LC** circuit inserted in parallel enclosed between 2 admittance inverters

# Practical implementations of impedance/admittance inverters

- Most often the quarter-wave transformer is used

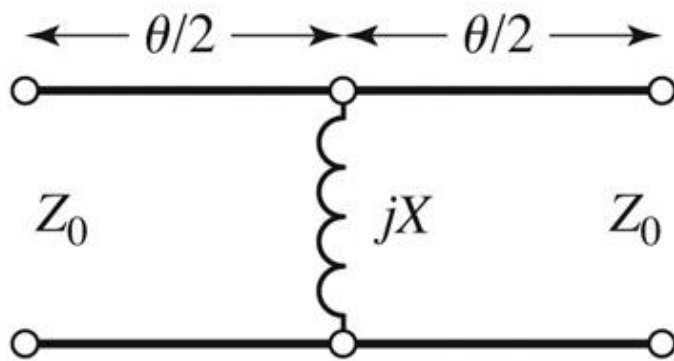


- Implementation with capacitor networks



# Practical implementations of impedance/admittance inverters

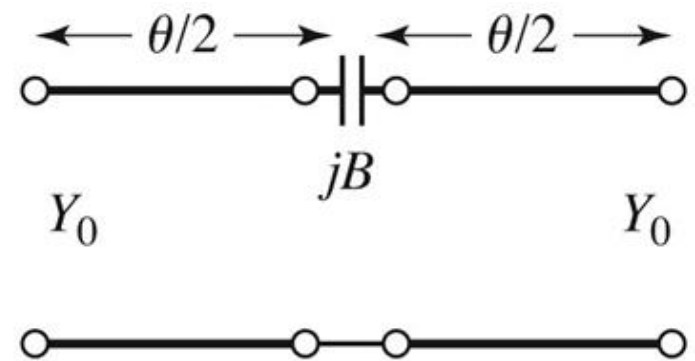
- Implementation with transmission lines and reactive elements



$$K = Z_0 \cdot \tan \left| \frac{\theta}{2} \right|$$

$$X = \frac{K}{1 - \left( \frac{K}{Z_0} \right)^2}$$

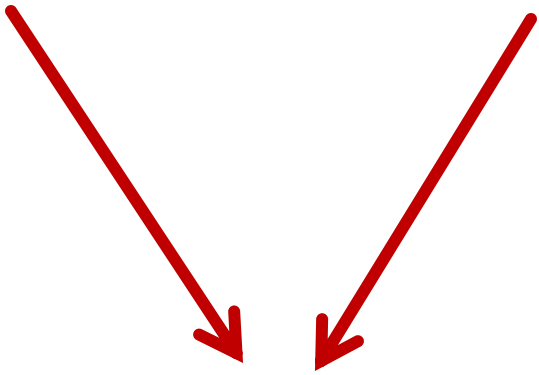
$$\theta = -\tan^{-1} \frac{2 \cdot X}{Z_0}$$



$$J = Y_0 \cdot \tan \left| \frac{\theta}{2} \right|$$

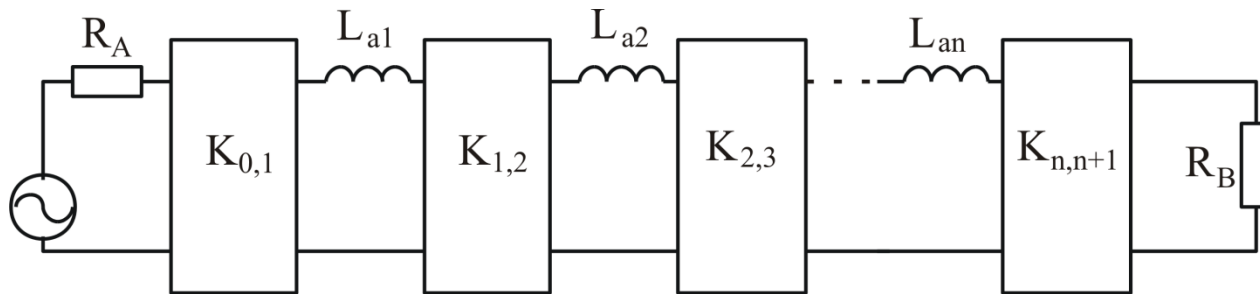
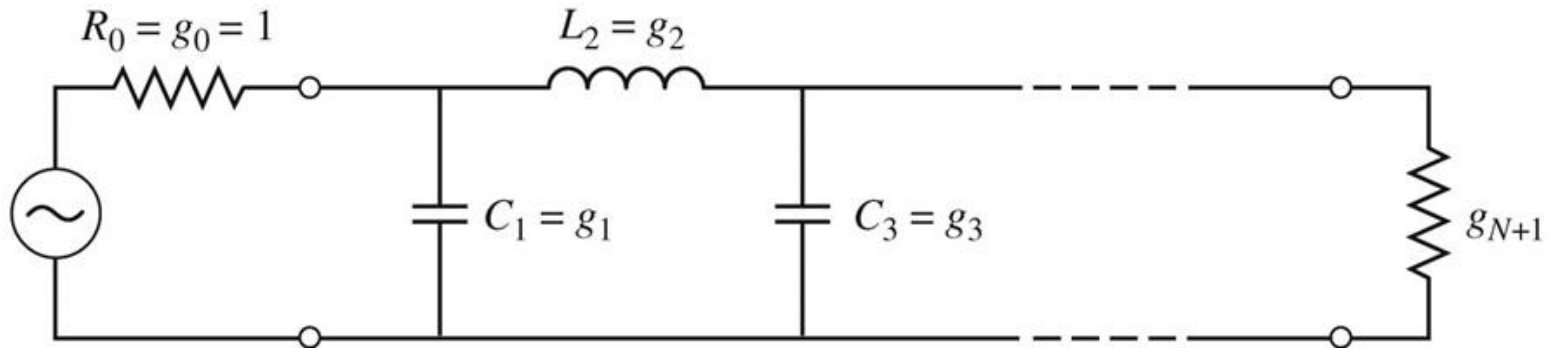
$$B = \frac{J}{1 - \left( \frac{J}{Y_0} \right)^2}$$

$$\theta = -\tan^{-1} \frac{2 \cdot B}{Y_0}$$


  
 $\theta < 0$

# Prototype filters using inverters

- Using impedance/admittance inverters we can implement prototype filters using a single type of reactive elements
  - Shunt C replaced by series L enclosed between 2 inverters



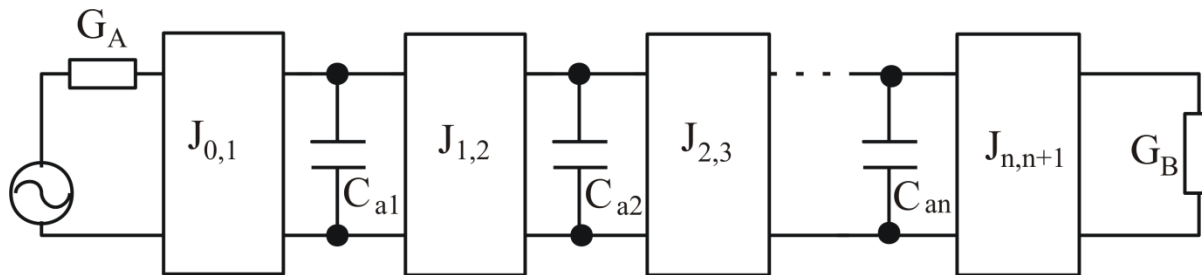
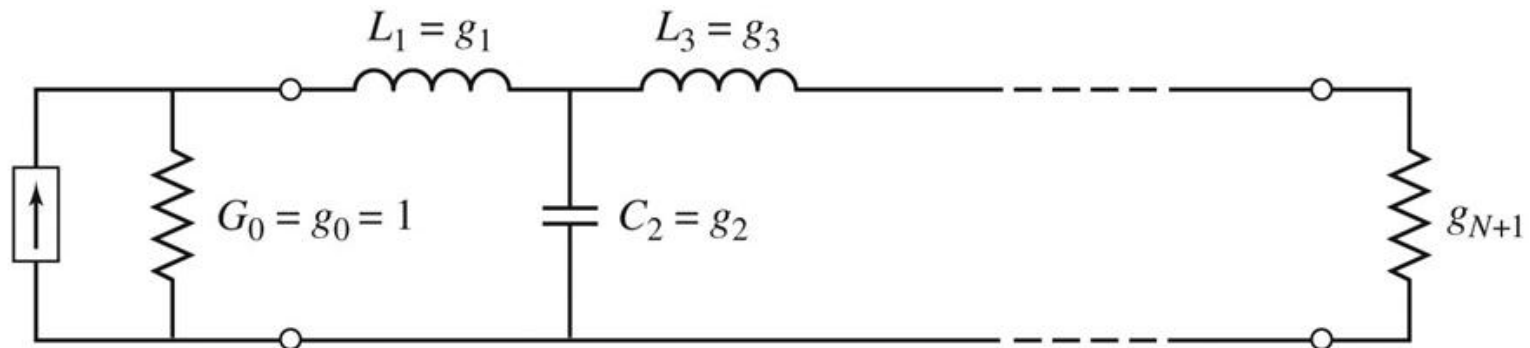
$$K_{0,1} = \sqrt{\frac{R_A \cdot L_{a,1}}{g_0 \cdot g_1}}$$

$$K_{k,k+1} \Big|_{k=1, n-1} = \sqrt{\frac{L_{a,k} \cdot L_{a,k+1}}{g_k \cdot g_{k+1}}}$$

$$K_{n,n+1} = \sqrt{\frac{L_{a,n} \cdot R_B}{g_n \cdot g_{n+1}}}$$

# Prototype filters using inverters

- Using impedance/admittance inverters we can implement prototype filters using a single type of reactive elements
  - Series L replaced by shunt C enclosed between 2 inverters



$$J_{0,1} = \sqrt{\frac{G_A \cdot C_{a,1}}{g_0 \cdot g_1}}$$

$$J_{k,k+1} \Big|_{k=1, n-1} = \sqrt{\frac{C_{a,k} \cdot C_{a,k+1}}{g_k \cdot g_{k+1}}}$$

$$J_{n,n+1} = \sqrt{\frac{C_{a,n} \cdot g_B}{g_n \cdot g_{n+1}}}$$

# Prototype filters using inverters

- For prototype filters using inverters formulas we have  $2 \cdot N + 1$  parameters and  $N + 1$  equations (to ensure the equivalence of the 2 schematics) so  $N$  parameters can be chosen freely
  - convenient values for the reactance can be chosen, and the required inverters will be computed from the equivalence equations or,
  - convenient inverters can be chosen, and the required reactance values will be computed from the equivalence equations

# BPF and BSF using inverters

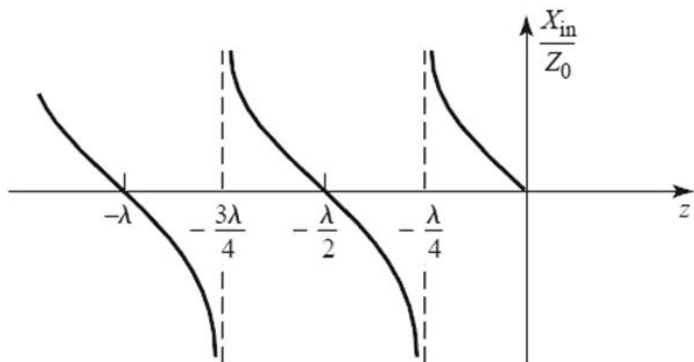
- The same principle can be applied to the BPF and BSF filters, those can be implemented using  $N+1$  inverters and  $N$  resonators (**series or shunt LC circuits** with resonant frequency  $\omega_o$ ) connected either in series or in parallel enclosed between 2 inverters
  - BPF are implemented with
    - series LC circuits connected in series between inverters
    - shunt LC circuits connected in parallel between inverters
  - BSF are implemented with
    - shunt LC circuits connected in series between inverters
    - series LC circuits connected in parallel between inverters

# Lines as resonators

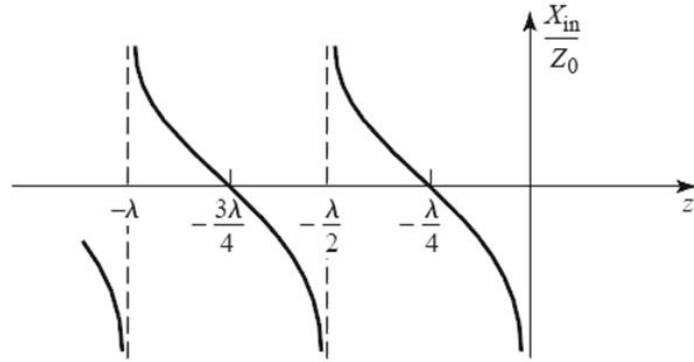
- The impedance of short-circuited or open-circuited line (stub) shows a resonant behavior that can be used to implement required resonators

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

$$Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l$$

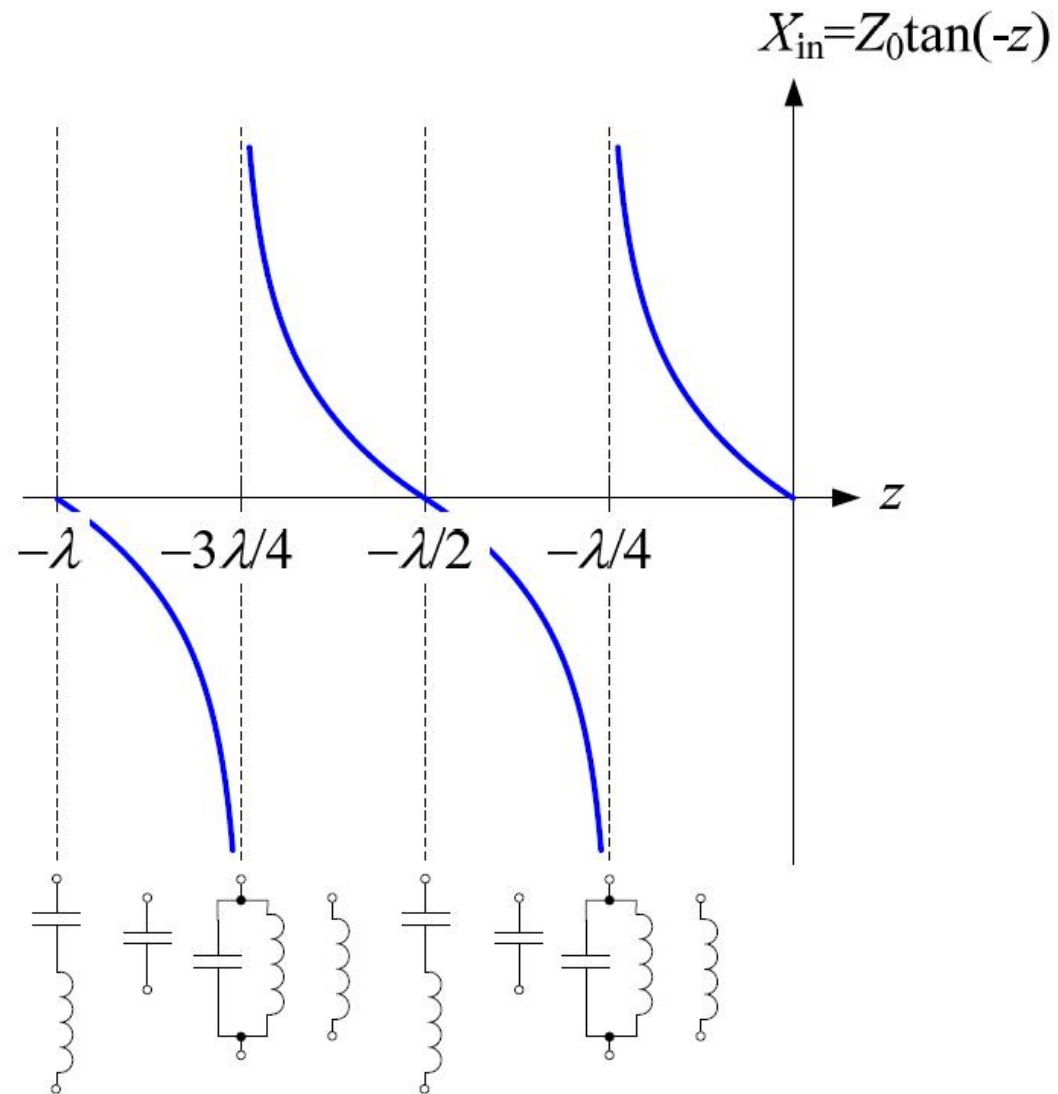


$$Z_{in,oc} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$



# Lines as resonators

- Short-circuited line
- For the frequency at which  $l = \lambda/4$  ( $\omega_0$ ) the line behaves as an shunt LC resonator circuit
  - the line shows capacitive behavior for lower frequencies ( $l > \lambda/4$ )
  - the line shows inductive behavior for higher frequencies ( $l < \lambda/4$ )
- Similar discussion for the open circuited line (equivalent to a series LC resonator around the frequency at which  $l = \lambda/4$ )



# BPF/BSP design formulas

- When the admittance inverters are implemented with quarter-wave transformers with  $Z_0$  characteristic impedance
  - BPF – short-circuited shunt stubs with  $l = \lambda/4$

$$Z_{0n} \approx \frac{\pi \cdot Z_0 \cdot \Delta}{4 \cdot g_n}$$

- BSF – open-circuited shunt stubs with  $l = \lambda/4$

$$Z_{0n} \approx \frac{4 \cdot Z_0}{\pi \cdot g_n \cdot \Delta}$$

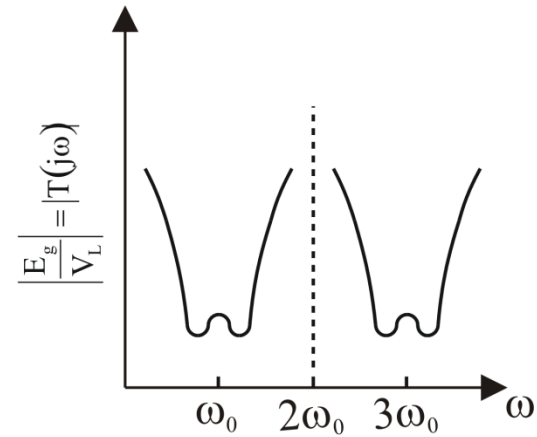
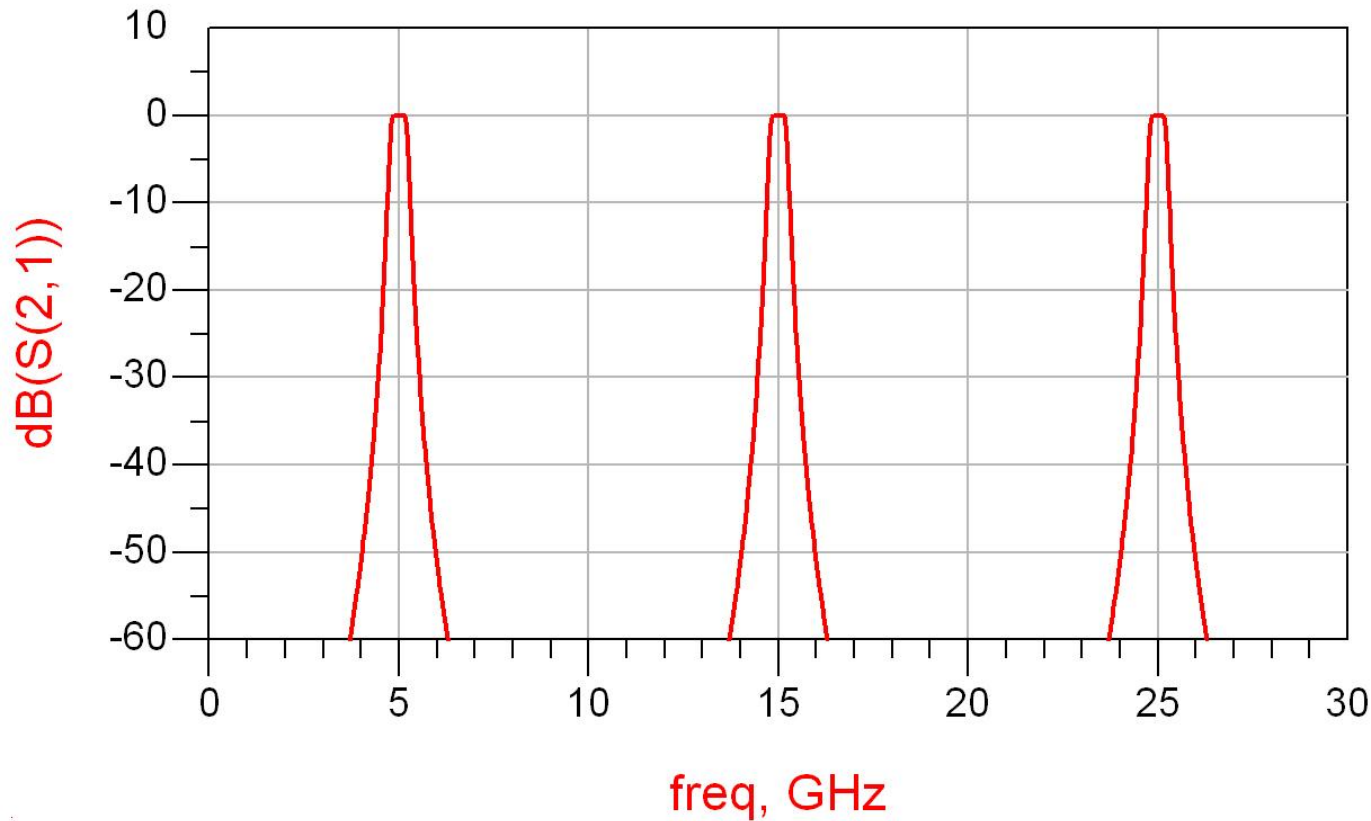
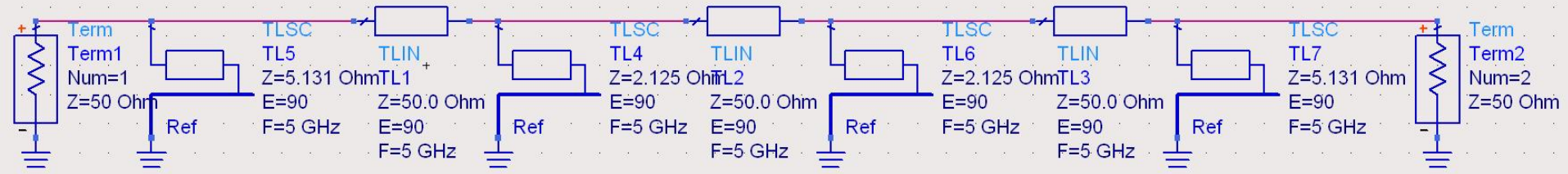
# Example

- Similar to a project assignment
- Follows the amplifier designed as in L8
- 4<sup>th</sup> order bandpass filter,  $f_0 = 5\text{GHz}$ , fractional bandwidth of the passband 10 %
- maximally flat table or formulas for  $g_n$ :

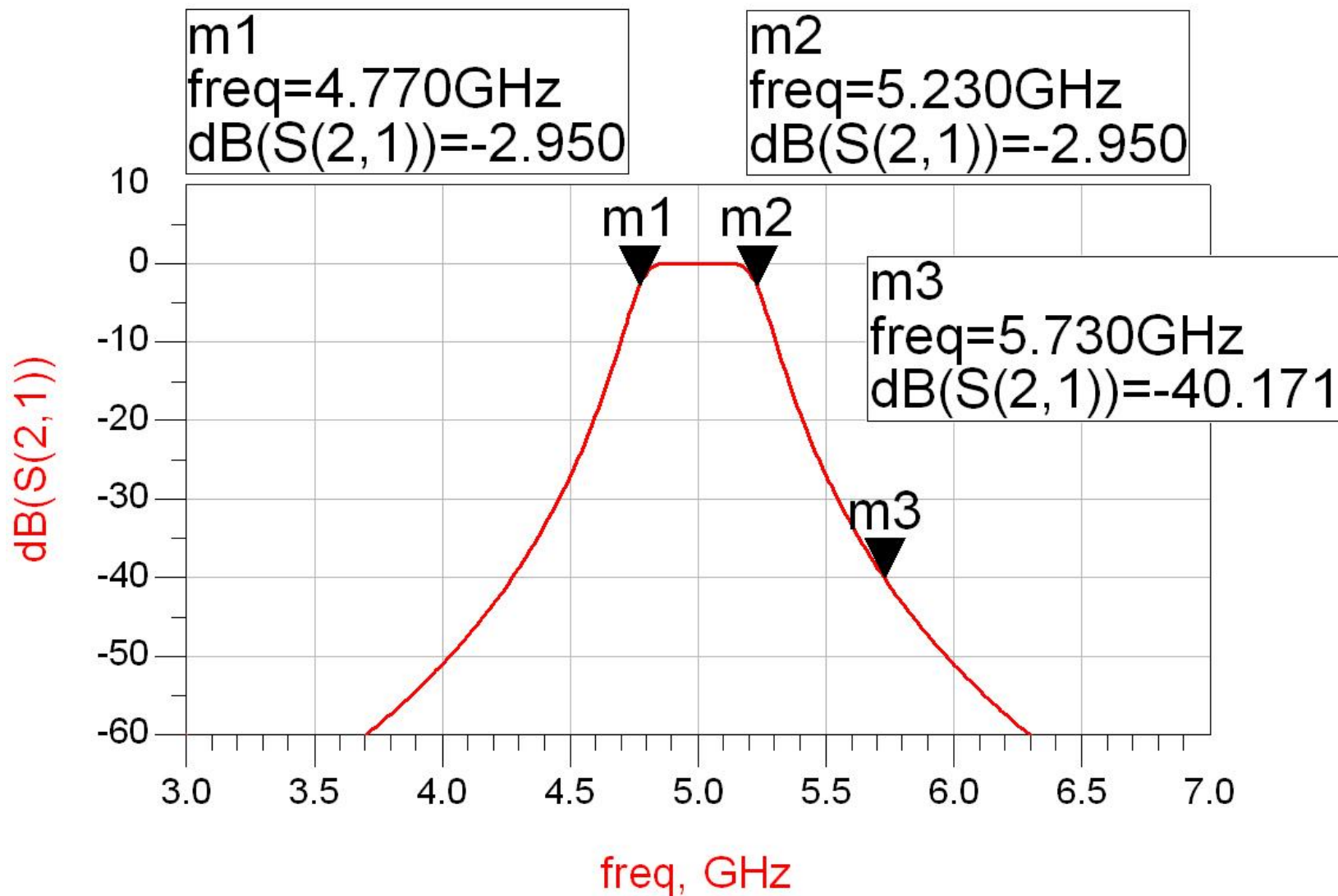
n	$g_n$	$Z_{0n}(\Omega)$
1	0.7654	5.131
2	1.8478	2.125
3	1.8478	2.125
4	0.7654	5.131

$$Z_{0n} \approx \frac{\pi \cdot Z_0 \cdot \Delta}{4 \cdot g_n}$$

# ADS – BPF

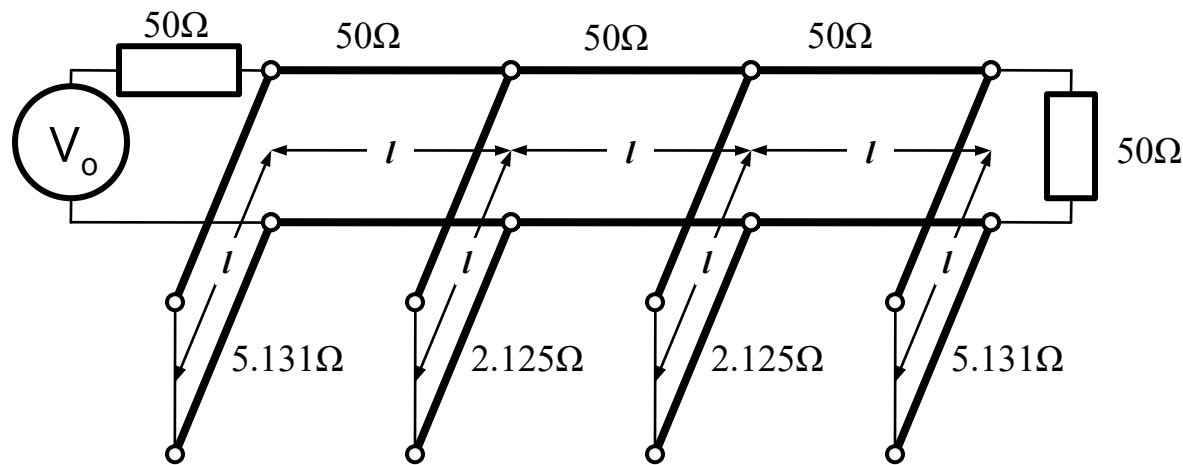


# ADS – BPF



# Example

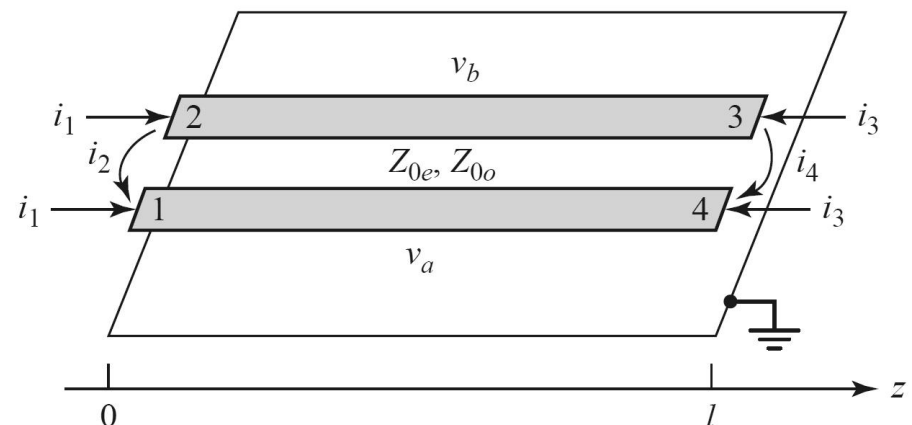
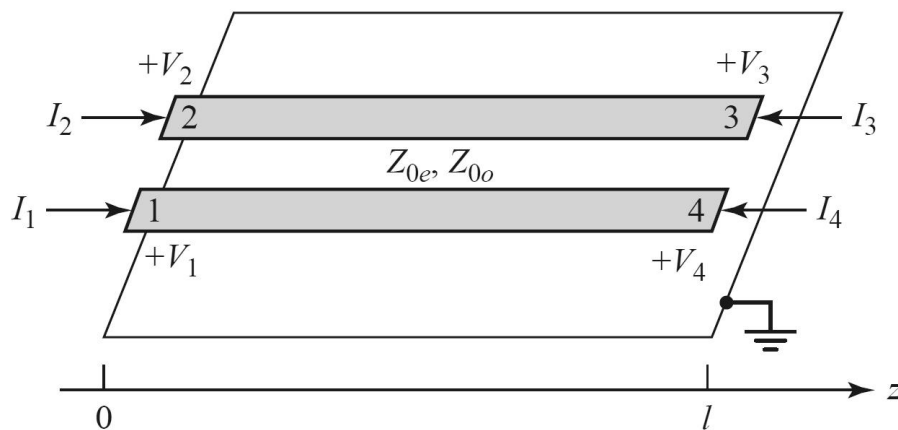
$$l = \frac{\lambda}{4} \Rightarrow \beta \cdot l = \frac{\pi}{2}$$



- Disadvantages of the filters using impedance inverters and lines as resonators:
  - short-circuited stubs (via-hole) for BPF
  - often the **characteristic impedances** for the stubs have **values** difficult to implement ( $2.125\Omega$ )

# Coupled Line Filters

- A parallel coupled line section model is obtained by even/odd mode analysis
- Even and odd modes are characterized by the characteristic even/odd mode impedances whose required values will impose the lines' geometry (width / distance between lines, depending on the line technology we use)



# Coupled Lines

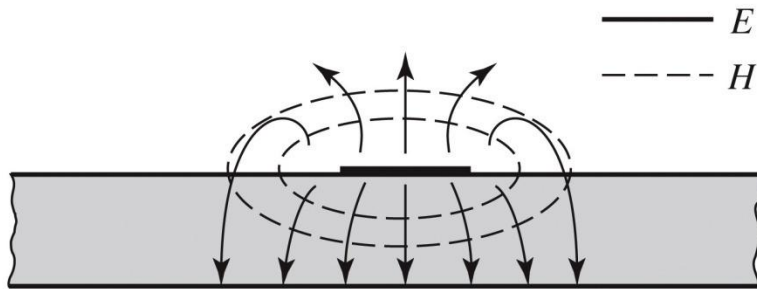
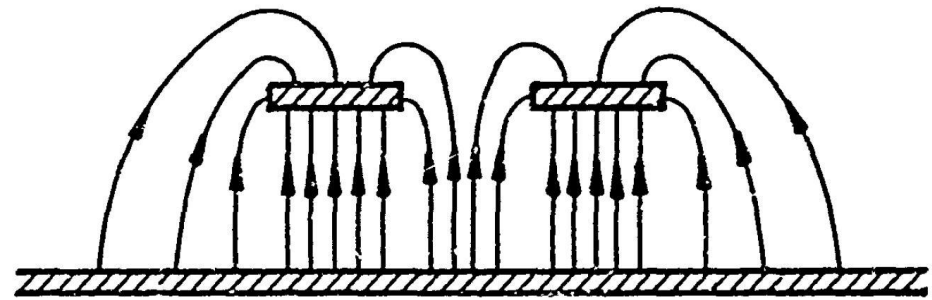
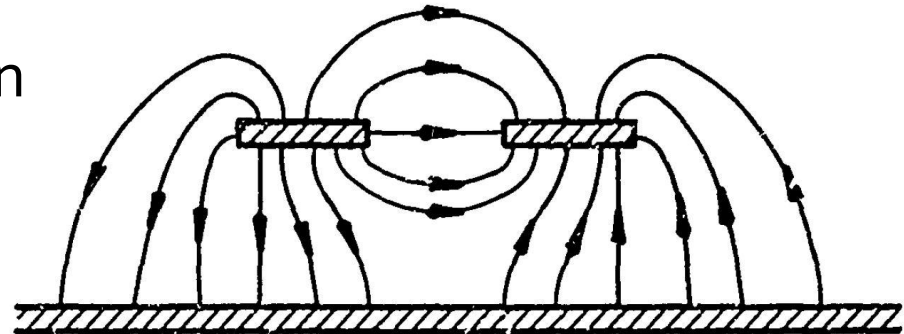


Figure 3.25b  
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b) EVEN MODE ELECTRIC FIELD PATTERN (SCHEMATIC)

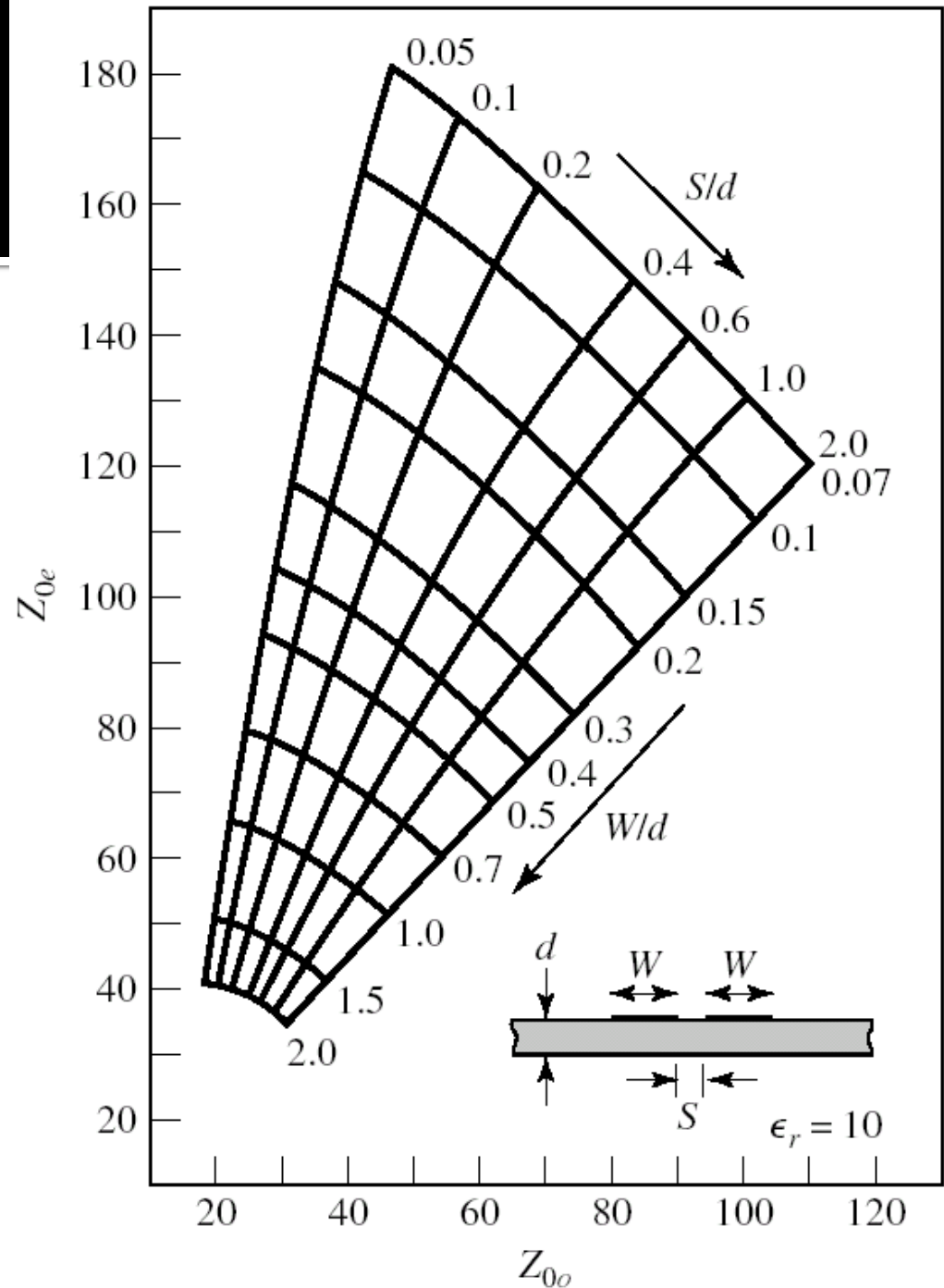


c) ODD MODE ELECTRIC FIELD PATTERN (SCHEMATIC)

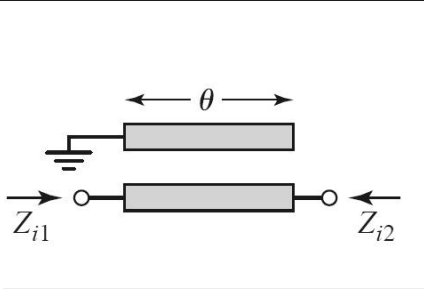
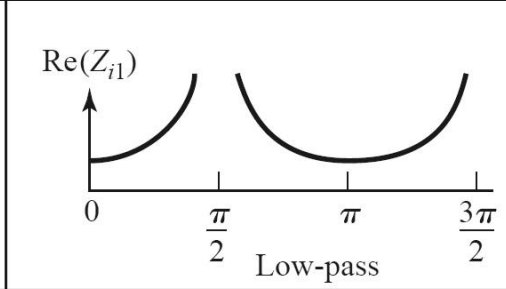
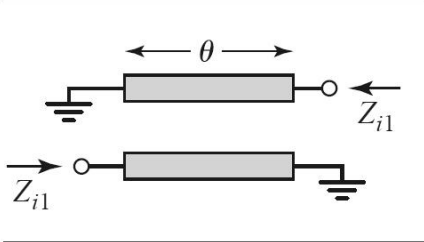
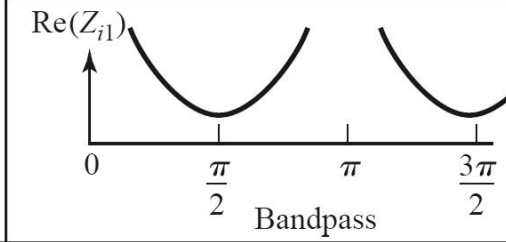
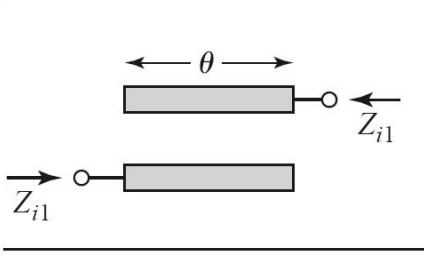
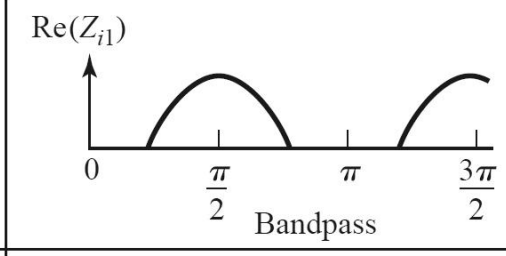
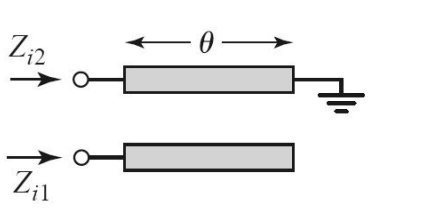
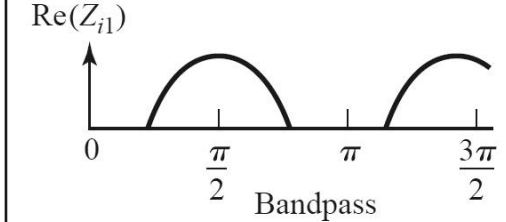
- Even mode - characterizes the common mode signal on the two lines
- Odd mode - characterizes the differential mode signal between the two lines
- Each of the two modes is characterized by **different** characteristic impedances

## Even- and odd-mode characteristic impedance

- Even- and odd-mode characteristic impedance design data for coupled microstrip lines on a substrate with  $\epsilon_r = 10$ .



# Coupled Line Filters

Circuit	Image Impedance	Response
	$Z_{i1} = \frac{2Z_{0e}Z_{0o} \cos \theta}{\sqrt{(Z_{0e} + Z_{0o})^2 \cos^2 \theta - (Z_{0e} - Z_{0o})^2}}$ $Z_{i2} = \frac{Z_{0e}Z_{0o}}{Z_{i1}}$	 <p style="text-align: center;">Low-pass</p>
	$Z_{i1} = \frac{2Z_{0e}Z_{0o} \sin \theta}{\sqrt{(Z_{0e} - Z_{0o})^2 - (Z_{0e} + Z_{0o})^2 \cos^2 \theta}}$	 <p style="text-align: center;">Bandpass</p>
	$Z_{i1} = \frac{\sqrt{(Z_{0e} - Z_{0o})^2 - (Z_{0e} + Z_{0o})^2 \cos^2 \theta}}{2 \sin \theta}$	 <p style="text-align: center;">Bandpass</p>
	$Z_{i1} = \frac{\sqrt{Z_{0e}Z_{0o}} \sqrt{(Z_{0e} - Z_{0o})^2 - (Z_{0e} + Z_{0o})^2 \cos^2 \theta}}{(Z_{0e} + Z_{0o}) \sin \theta}$ $Z_{i2} = \frac{Z_{0e}Z_{0o}}{Z_{i1}}$	 <p style="text-align: center;">Bandpass</p>



# Coupled Line Filters

- Bandpass filter with resonance at  $\theta = \pi/2$  ( $l = \lambda/4$ )

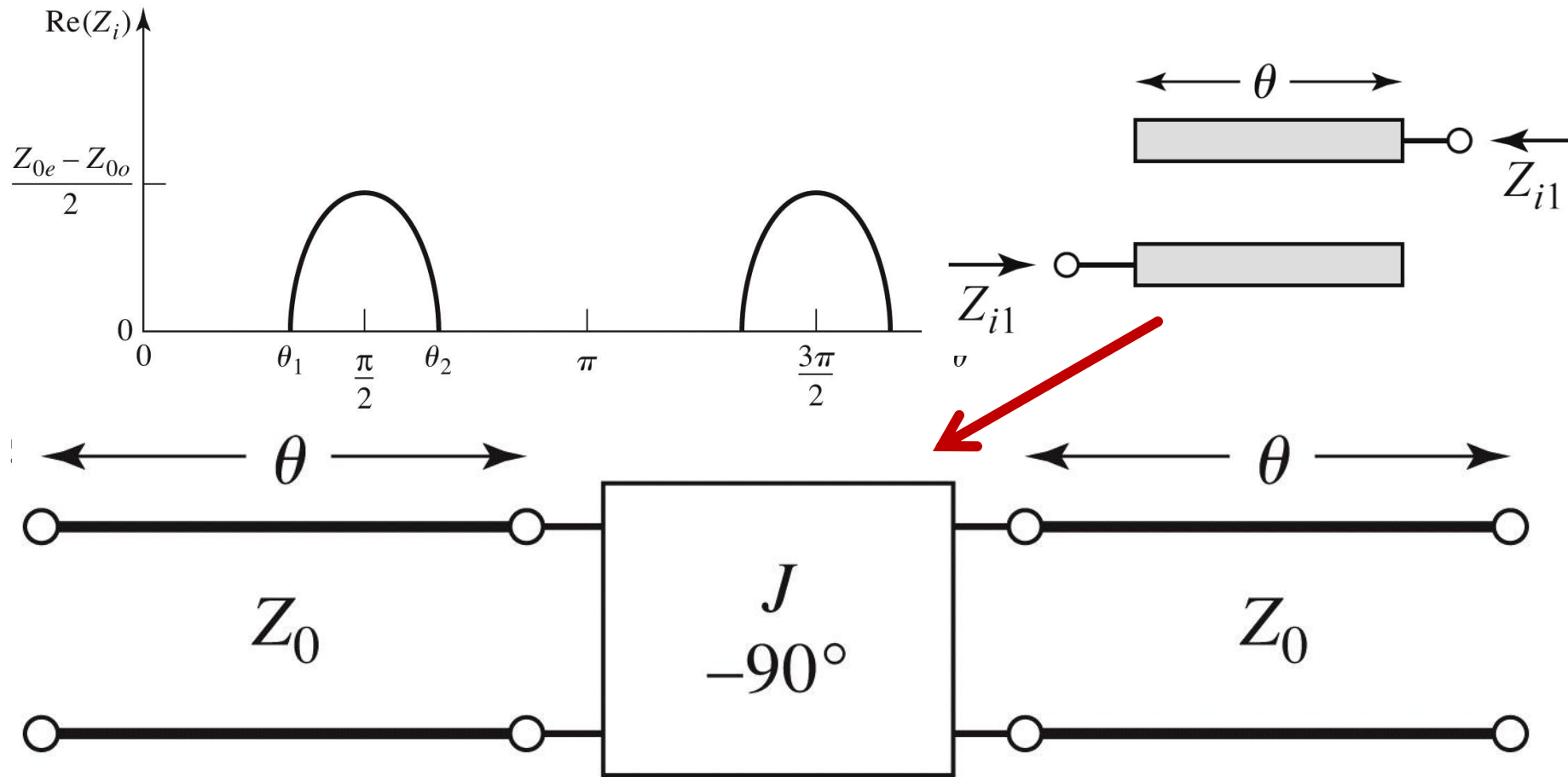
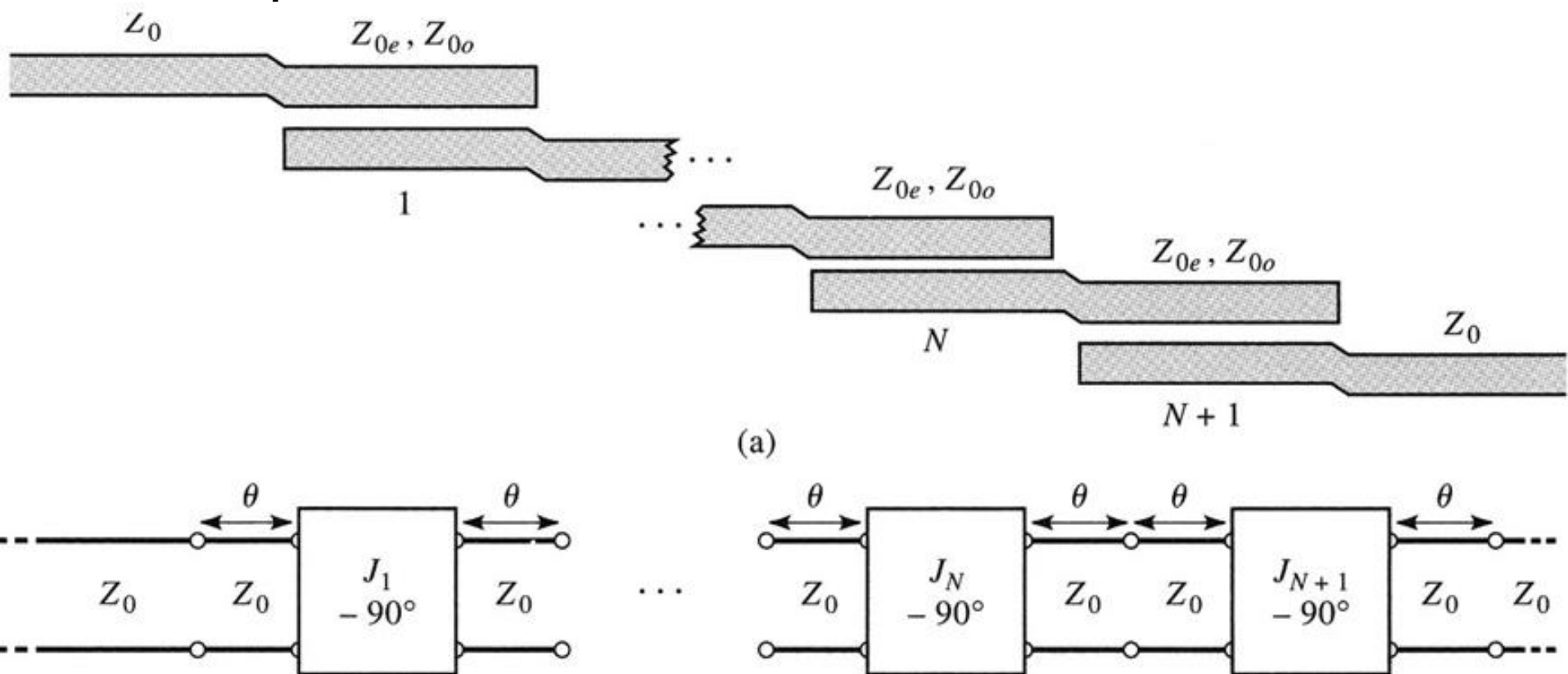


Figure 8.44

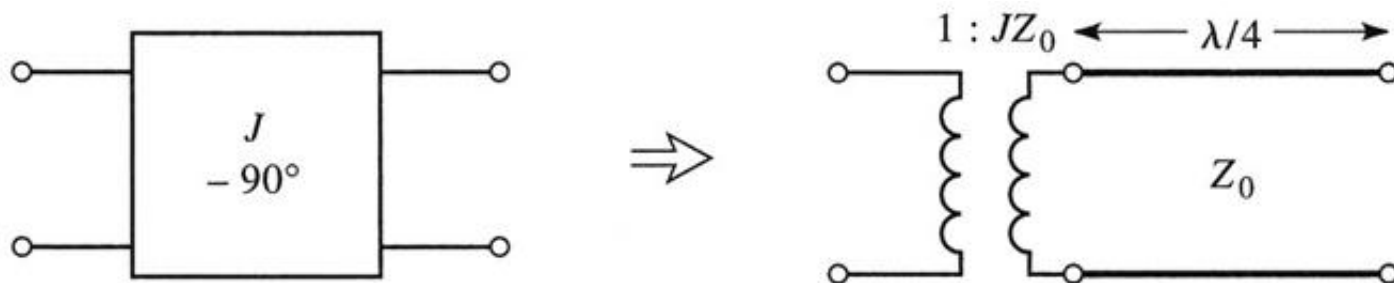
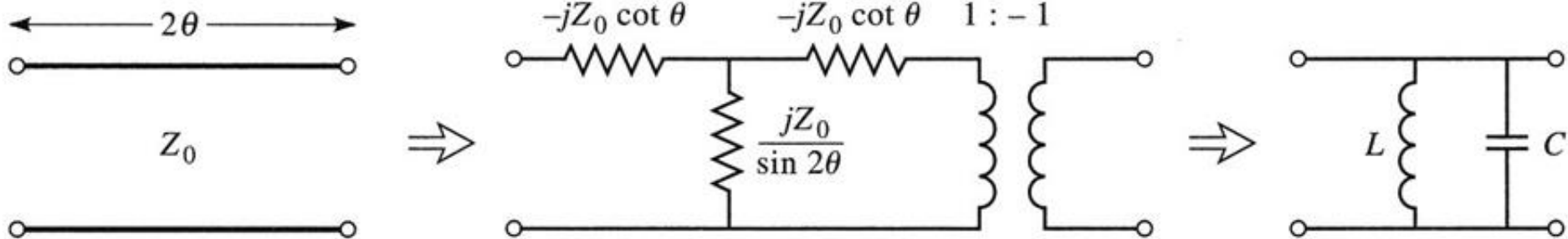
# Coupled Line Filters

- We get a  $N^{\text{th}}$  order filter with  $N+1$  parallel coupled line section



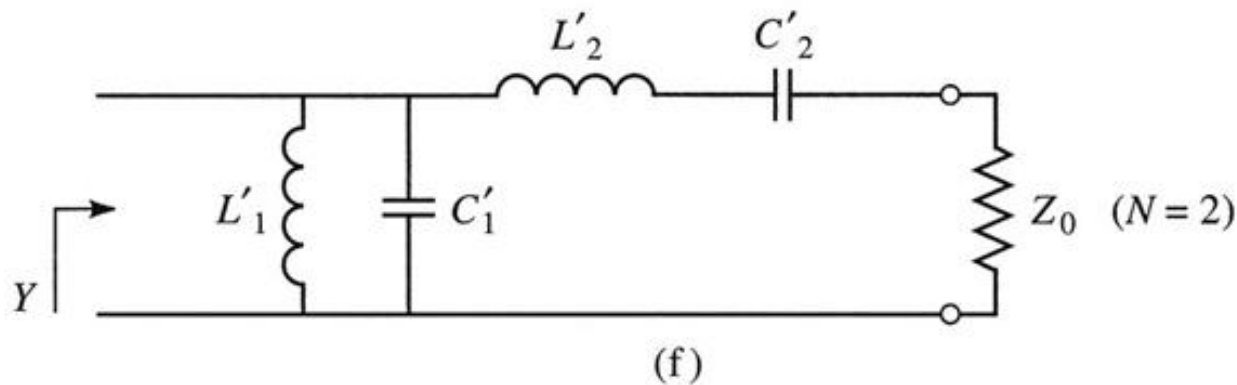
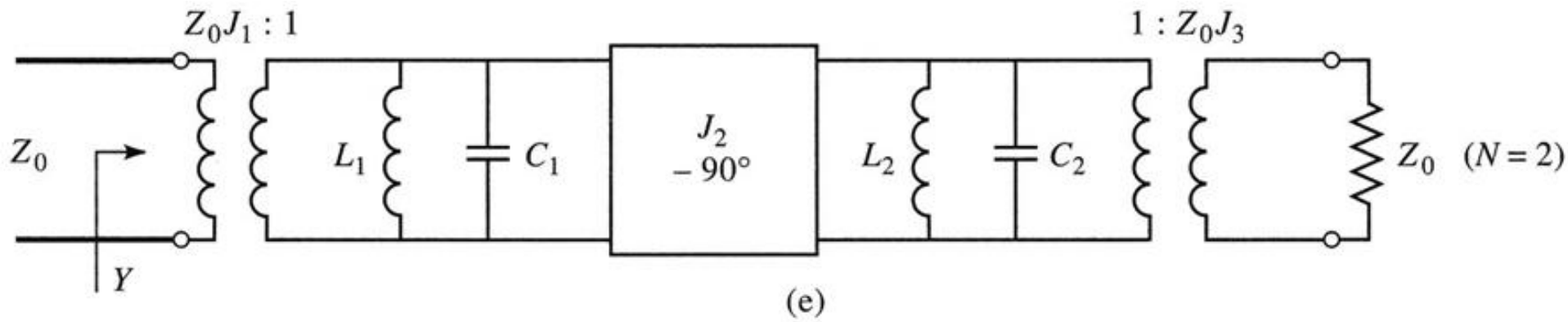
# Coupled Line Filters

- Equivalent circuits for
  - transmission lines of length  $2\theta$
  - admittance inverters



# Coupled Line Filters $N=2$

- We get a 2<sup>nd</sup> order BPF behavior with 3 coupled lines sections



# Coupled Line Filters design formulas

- Compute the inverters from prototype parameters

$$Z_0 \cdot J_1 = \sqrt{\frac{\pi \cdot \Delta}{2 \cdot g_1}} \quad Z_0 \cdot J_n = \frac{\pi \cdot \Delta}{2 \cdot \sqrt{g_{n-1} \cdot g_n}}, n = \overline{2, N} \quad Z_0 \cdot J_{N+1} = \sqrt{\frac{\pi \cdot \Delta}{2 \cdot g_N \cdot g_{N+1}}}$$

- Compute coupled line parameters  $Z_{oe}/Z_{oo}$  (all of length  $l = \lambda/4$ )

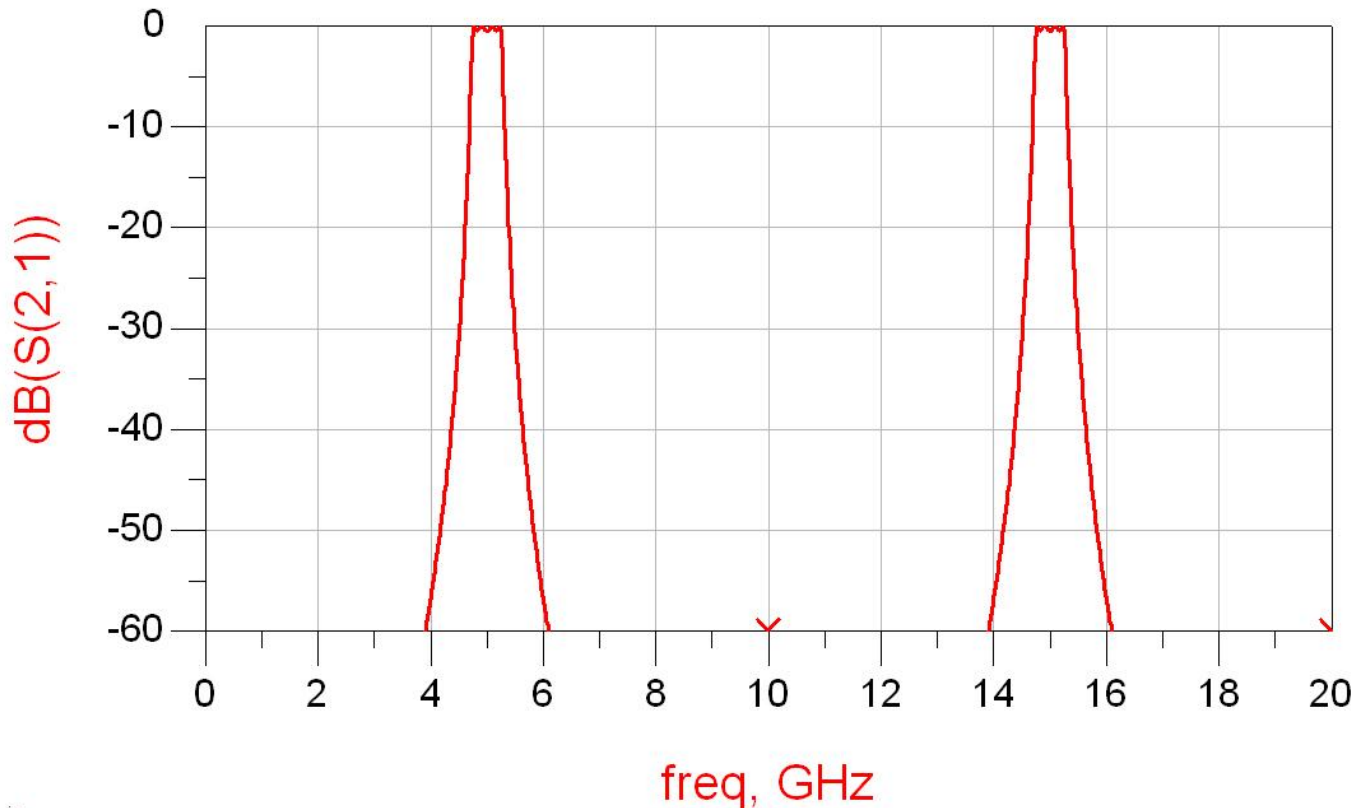
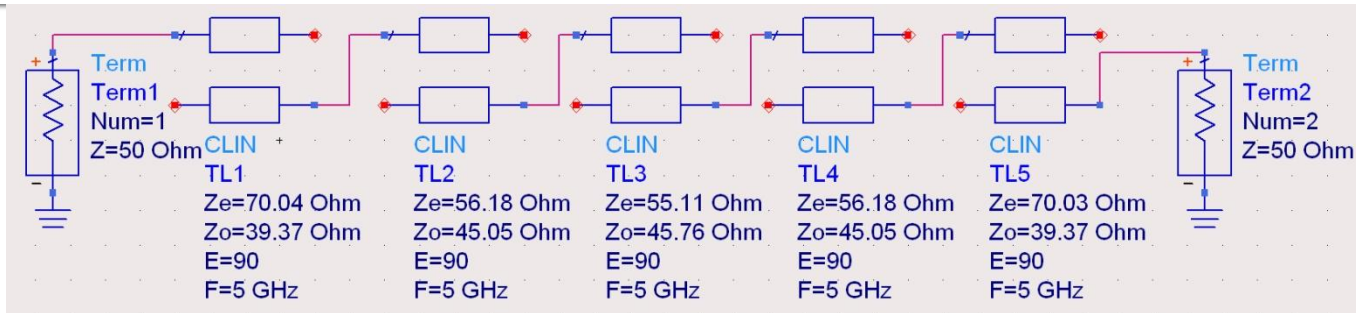
$$\begin{aligned} Z_{oe,n} &= Z_0 \cdot \left[ 1 + J_n \cdot Z_0 + (J_n \cdot Z_0)^2 \right] \\ Z_{oo,n} &= Z_0 \cdot \left[ 1 - J_n \cdot Z_0 + (J_n \cdot Z_0)^2 \right] \end{aligned} \quad n = \overline{1, N+1}$$

# Example

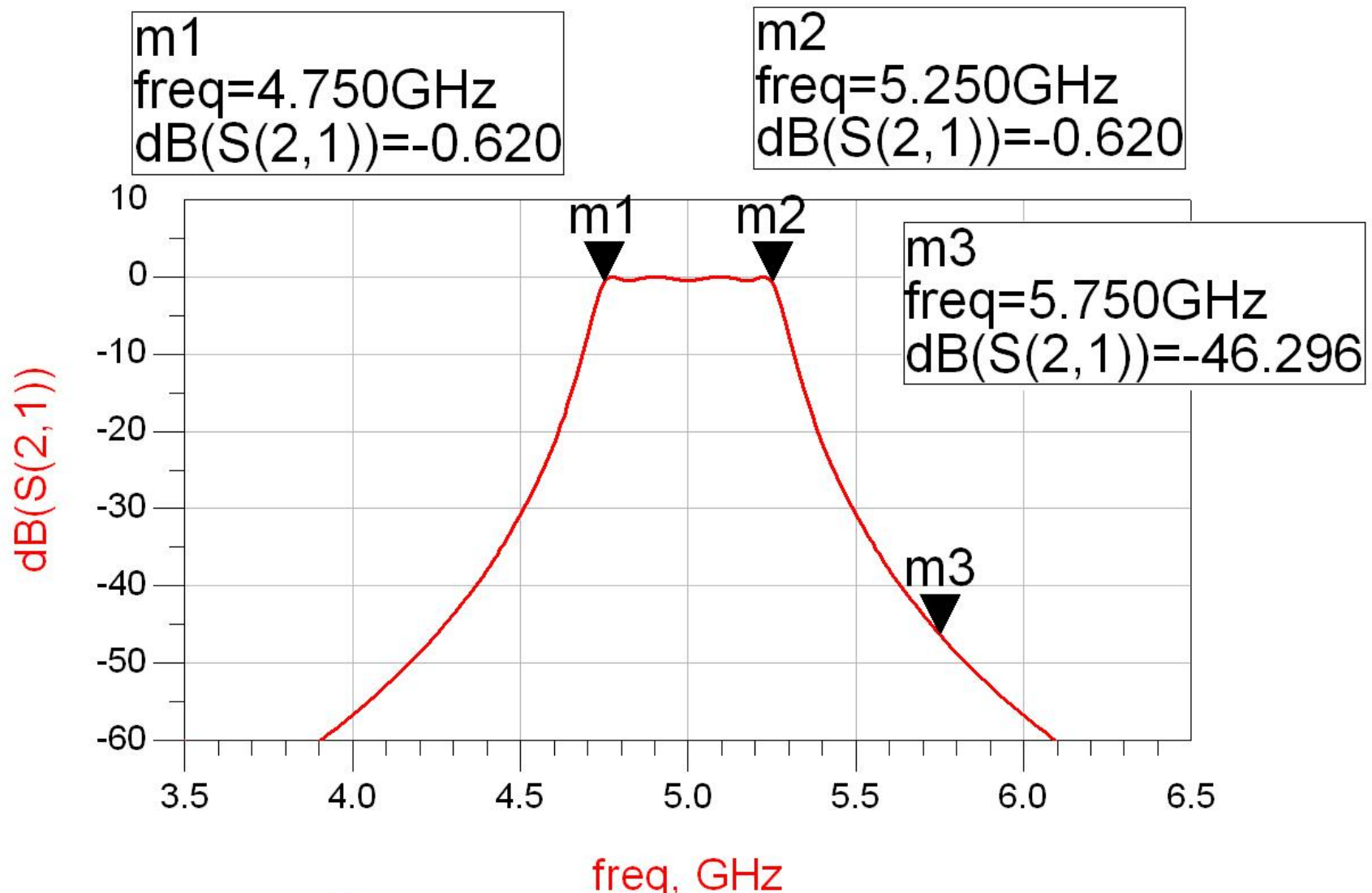
- Similar to a project assignment
- Follows the amplifier designed as in L10
- **4<sup>th</sup>** order bandpass filter,  $f_0 = 5\text{GHz}$ , fractional bandwidth of the passband 10 %
- 0.5dB equal-ripple table for  $g_n$  followed by filter design formulas

n	g	ZoJn	Zoe	Zoo
1	1.6703	0.306664	70.04	39.37
2	1.1926	0.111295	56.18	45.05
3	2.3661	0.09351	55.11	45.76
4	0.8419	0.111294	56.18	45.05
<b>5</b>	1.9841	0.306653	70.03	39.37

# ADS – coupled line BPF



# ADS – coupled line BPF



# Examples

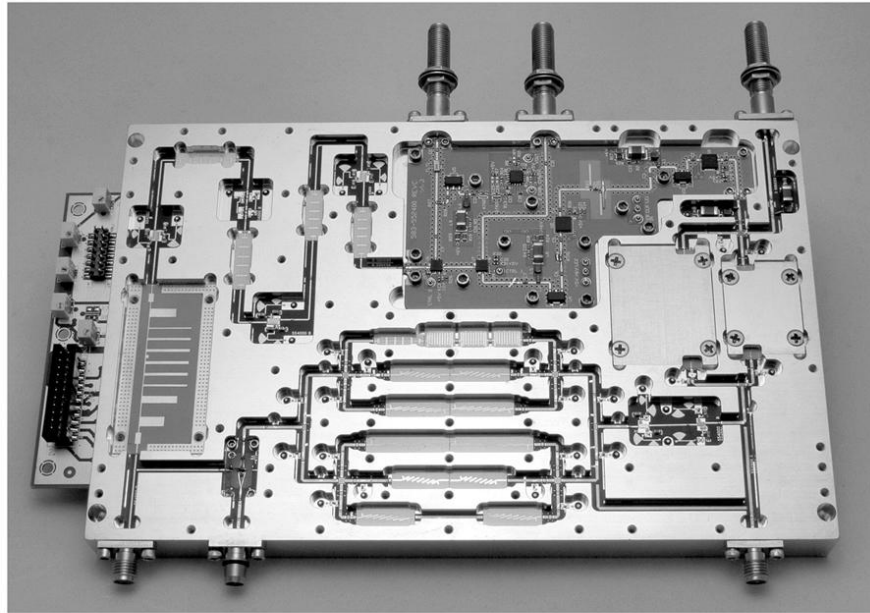
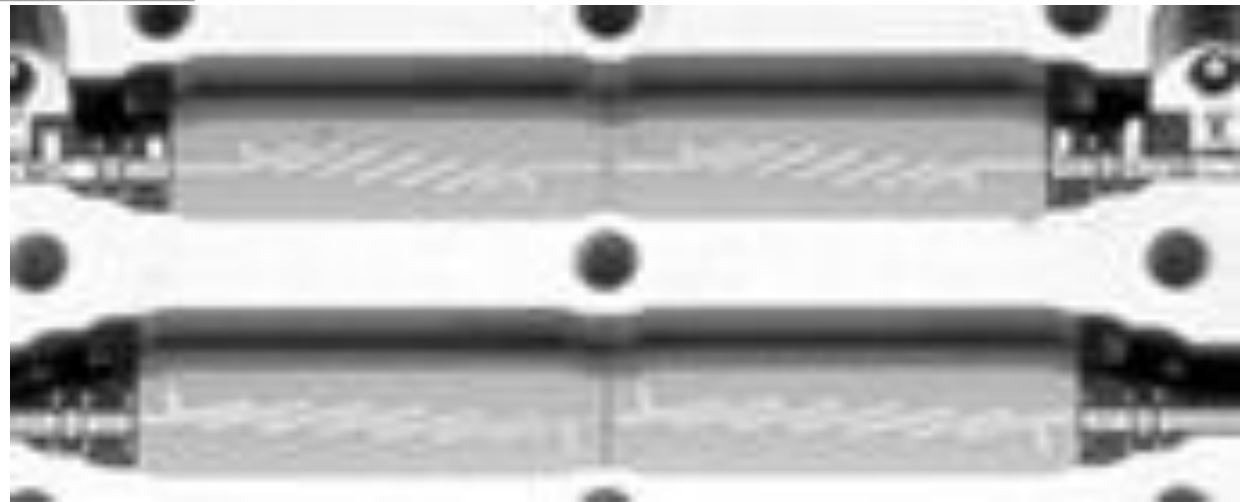
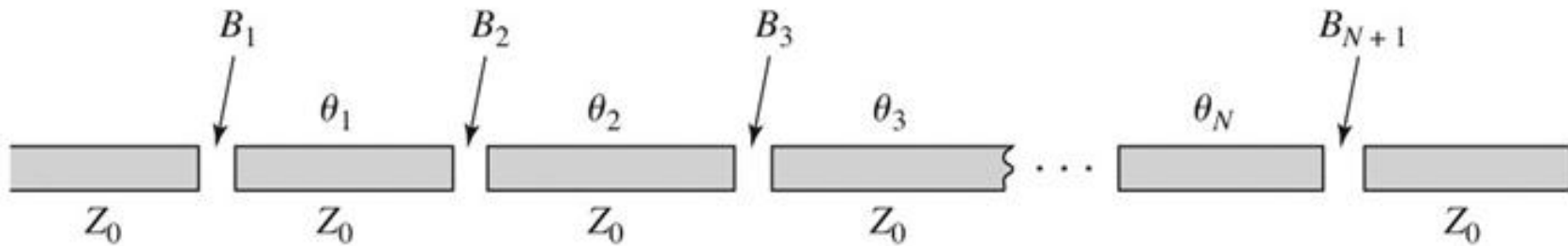


Figure 8.55  
Courtesy of LNX Corporation, Salem, N.H.

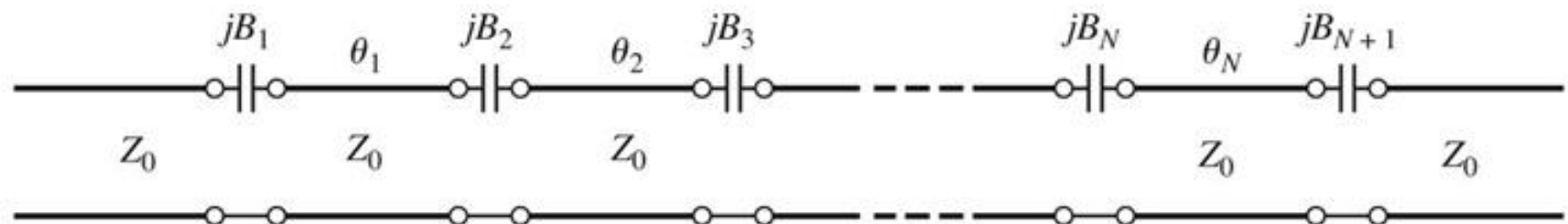


# Bandpass Filters Using Capacitively Coupled Series Resonators

- The gaps between the resonators ( $\sim\lambda/2$ ) generate a capacitive coupling between two resonators and can be approximated as series capacitors



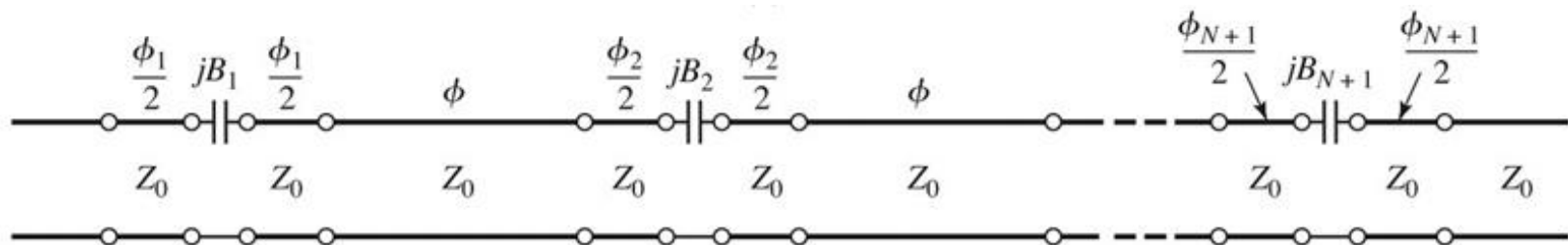
(a)



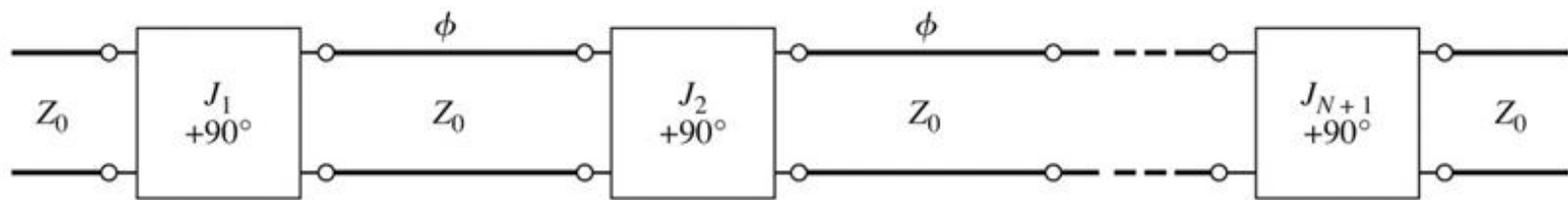
(b)

# Bandpass Filters Using Capacitively Coupled Series Resonators

- From the real physical length of the resonators, some part is used implement a admittance inverter (the remainder  $\phi = \pi$ ,  $l = \lambda/2$ , resonator)



(c)



(d)

Figure 8.50

# Bandpass Filters Using Capacitively Coupled Series Resonators design

- Compute the inverters (similar to coupled lines)

$$Z_0 \cdot J_1 = \sqrt{\frac{\pi \cdot \Delta}{2 \cdot g_1}} \quad Z_0 \cdot J_n = \frac{\pi \cdot \Delta}{2 \cdot \sqrt{g_{n-1} \cdot g_n}}, n = \overline{2, N} \quad Z_0 \cdot J_{N+1} = \sqrt{\frac{\pi \cdot \Delta}{2 \cdot g_N \cdot g_{N+1}}}$$

- Compute capacitive susceptances

$$B_n = \frac{J_n}{1 - (Z_0 \cdot J_n)^2}, n = \overline{1, N+1}$$

- Compute the line lengths that must be “borrowed” to implement the inverters

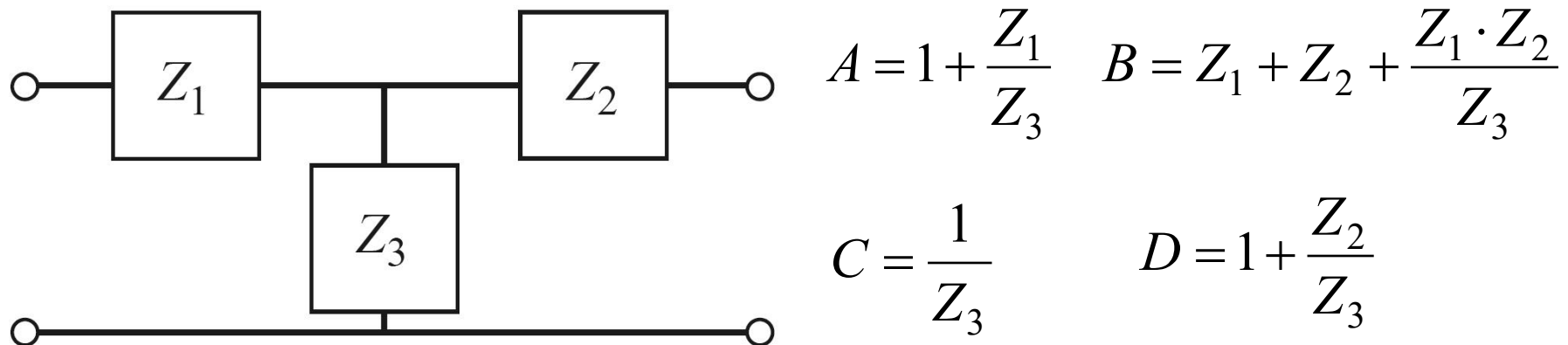
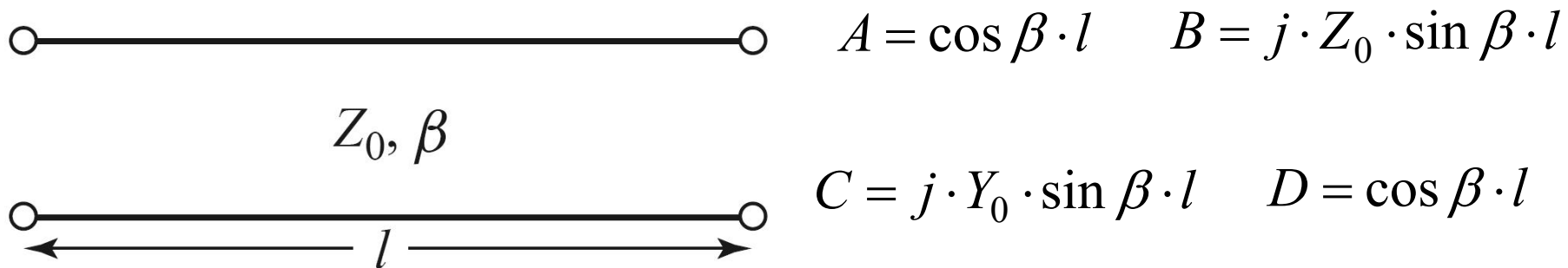
$$\phi_n = -\tan^{-1}(2 \cdot Z_0 \cdot B_n), n = \overline{1, N+1} \quad \phi_n < 0, n = \overline{1, N+1}$$

- Compute the actual length of the lines ( $\lambda/2 + \text{borr.}$ )

$$\theta_i = \pi + \frac{1}{2} \cdot (\phi_i + \phi_{i+1}) = \pi - \frac{1}{2} \cdot [\tan^{-1}(2 \cdot Z_0 \cdot B_i) + \tan^{-1}(2 \cdot Z_0 \cdot B_{i+1})], i = \overline{1, N}$$

# Equivalent circuits for short sections of transmission lines

- ABCD matrix (L5)
- short line, model with lumped elements is valid



# Equivalent circuits for short sections of transmission lines

- The shunt element is capacitive

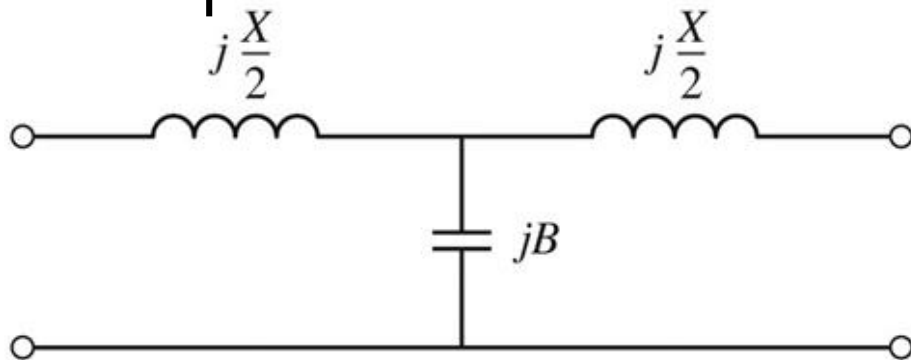
$$Z_3 = \frac{1}{j \cdot Y_0 \cdot \sin \beta \cdot l}$$

- Series elements are equal, and inductive

$$\cos \beta \cdot l = 1 + \frac{Z_1}{Z_3} = 1 + \frac{Z_2}{Z_3}$$

$$Z_1 = Z_2 = Z_3 \cdot (\cos \beta \cdot l - 1) = -j \cdot Z_0 \cdot \frac{\cos \beta \cdot l - 1}{\sin \beta \cdot l} = j \cdot Z_0 \cdot \tan \frac{\beta \cdot l}{2}$$

- Equivalent circuit



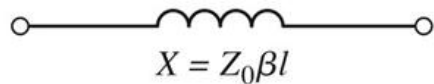
$$\frac{X}{2} = Z_0 \cdot \tan \frac{\beta \cdot l}{2}$$

$$B = \frac{1}{Z_0} \cdot \sin \beta \cdot l$$

# Equivalent circuits for short sections of transmission lines

- depending on the characteristic impedance:

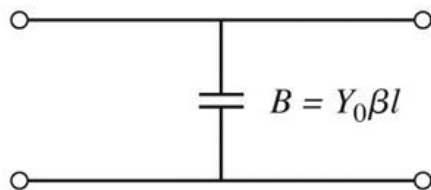
- high  $Z_0 \gg$



$$X \cong Z_0 \cdot \beta \cdot l \quad \beta \cdot l < \frac{\pi}{4} \quad Z_0 = Z_h$$



- low  $Z_0 \ll$



$$B \cong Y_0 \cdot \beta \cdot l \quad \beta \cdot l < \frac{\pi}{4} \quad Z_0 = Z_l$$

# Stepped-impedance low-pass filter

- Series L, shunt C, we realize low-pass filters
- We use

- lines with high characteristic impedance to implement an series inductor

$$\beta \cdot l = \frac{L \cdot R_0}{Z_h}$$

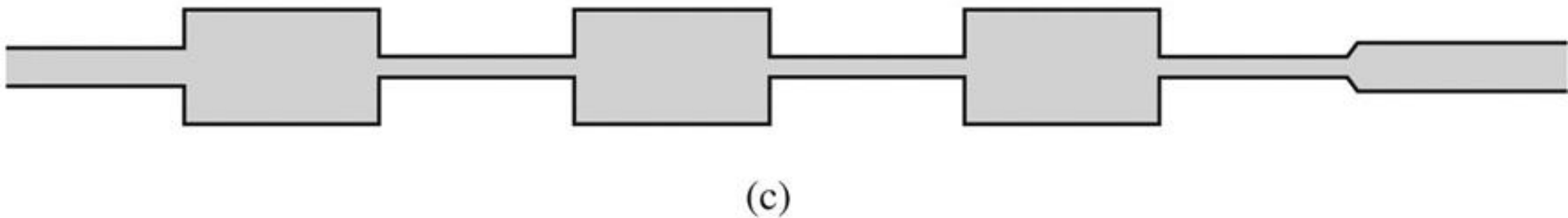
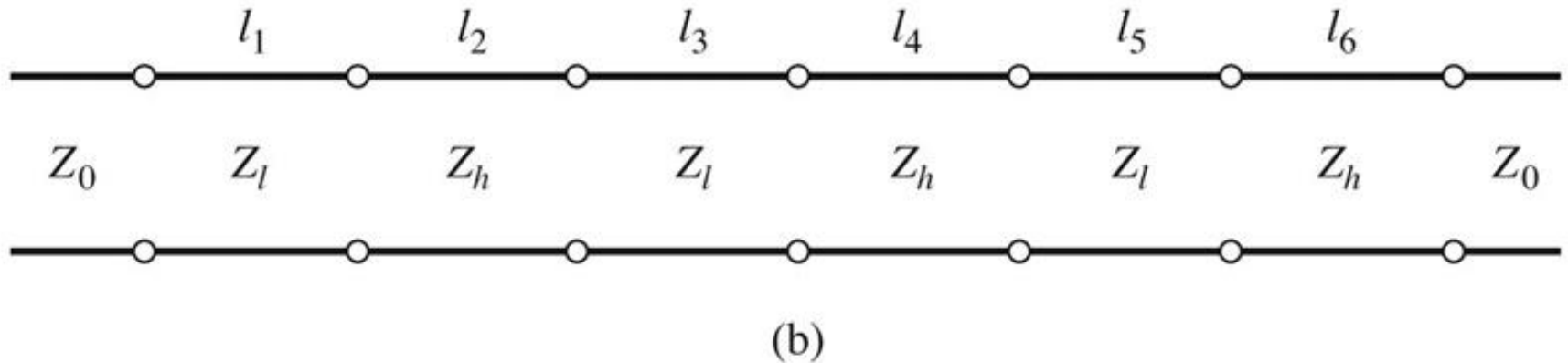
- lines with low characteristic impedance to implement a shunt capacitor

$$\beta \cdot l = \frac{C \cdot Z_l}{R_0}$$

- usually the highest and lowest characteristic impedance that can be practically fabricated

# Stepped-impedance LPF

- Not all the lines will result with the same length so the filter response is not periodic in frequency

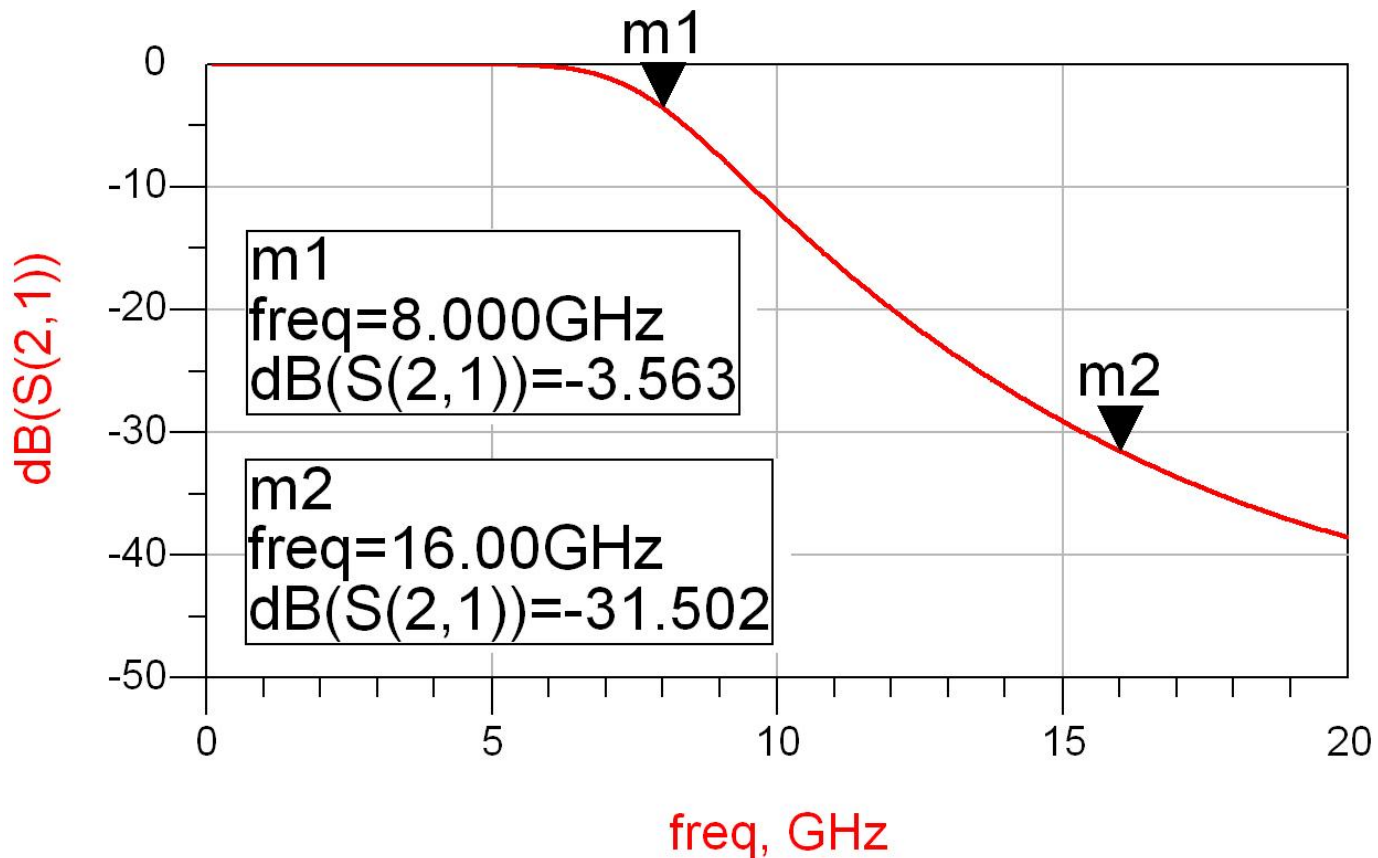
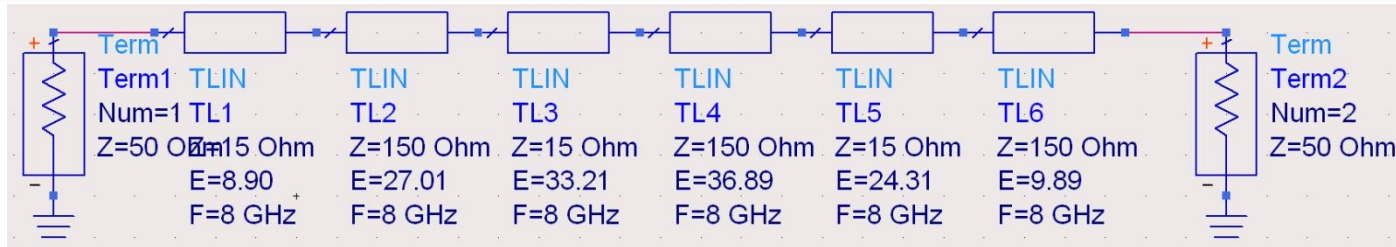


# Example

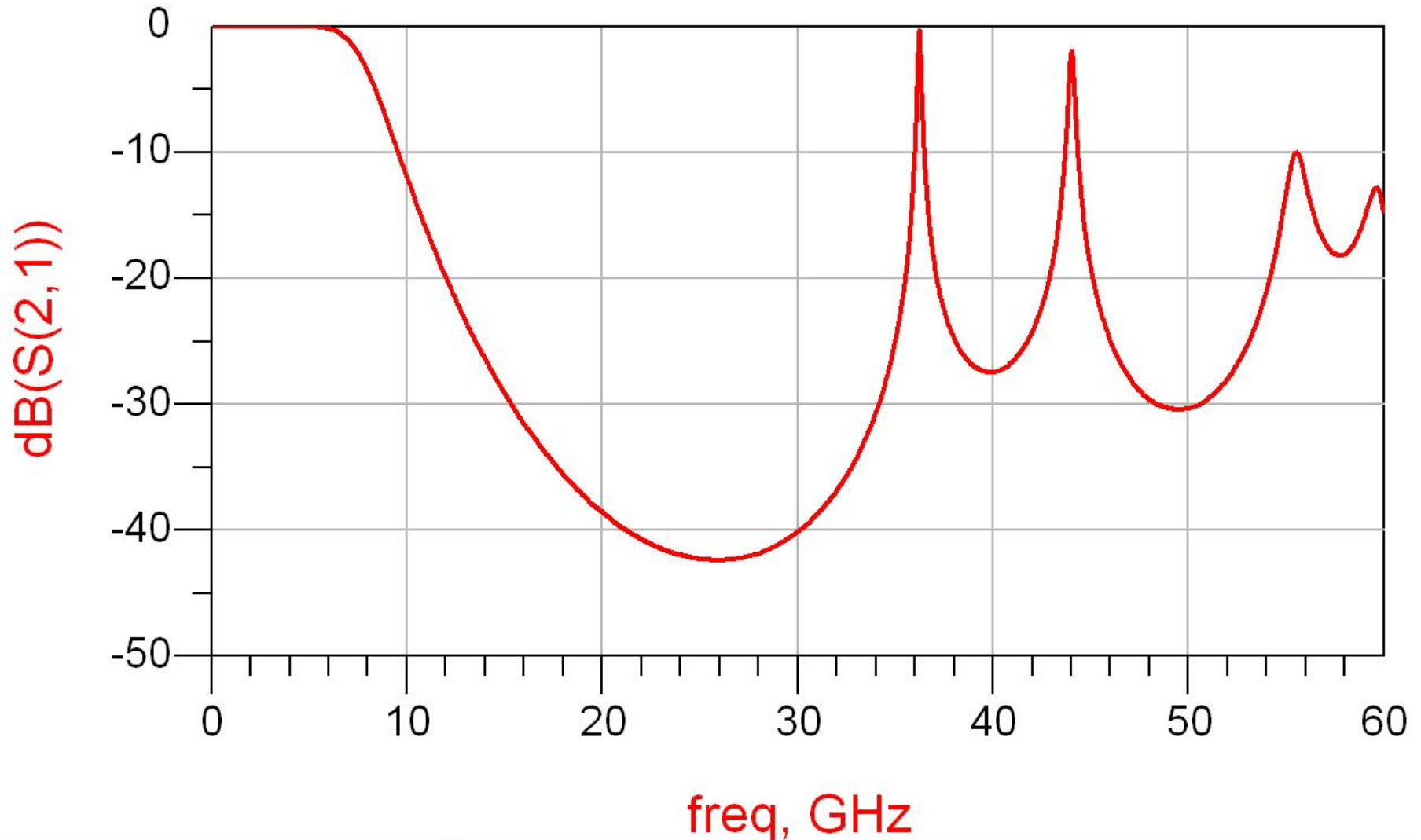
- LPF with 8GHz cutoff frequency, 6<sup>th</sup> order. Maximum realizable impedance is 150 $\Omega$  and lowest 15 $\Omega$ .

n	$g_n$	L/C <sub>n</sub>	Z	$\theta_n$ [rad]	$\theta_n$ [°]
1	0.5176	0.206pF	15	0.155	8.90
2	1.4142	1.407nH	150	0.471	27.01
3	1.9318	0.769pF	15	0.580	33.21
4	1.9318	1.922nH	150	0.644	36.89
5	1.4142	0.563pF	15	0.424	24.31
6	0.5176	0.515nH	150	0.173	9.89

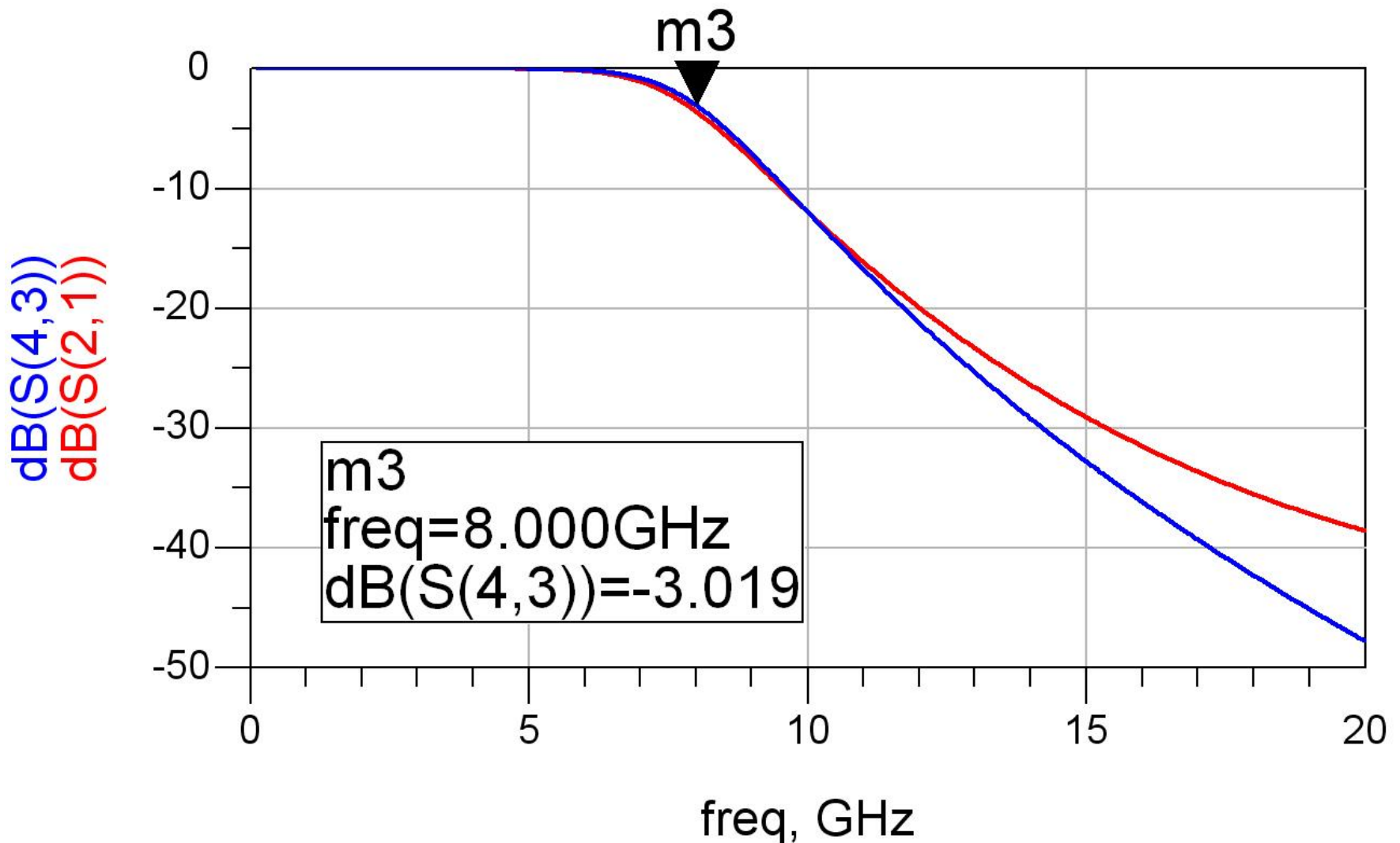
# ADS – Stepped-impedance LPF



# ADS – Stepped-impedance LPF



# ADS – Stepped-impedance LPF – compared with lumped elements



# Examples

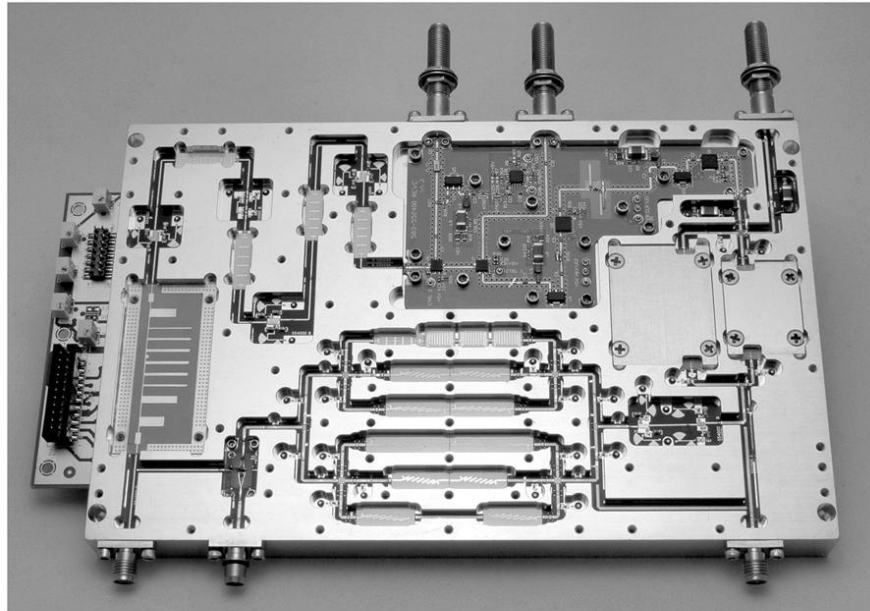
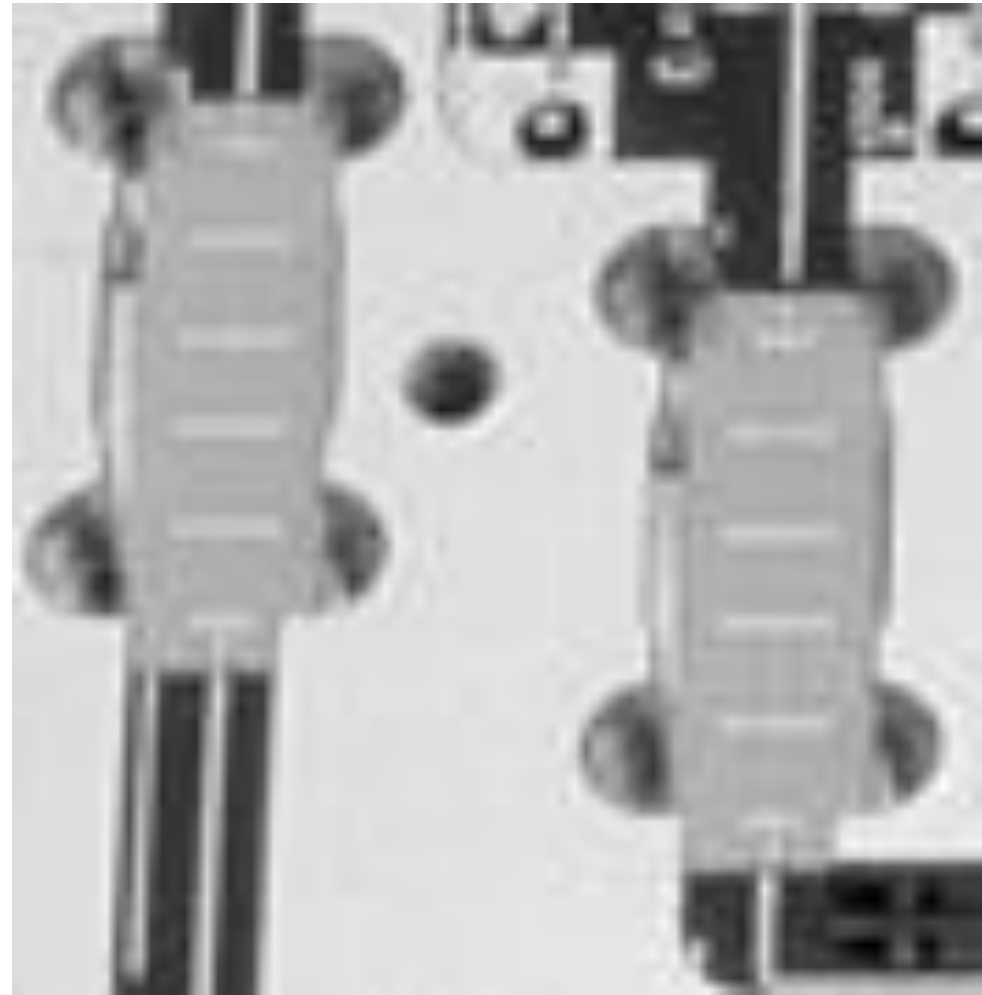


Figure 8.55  
Courtesy of LNX Corporation, Salem, N.H.



# Contact

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